Multiwire Thermocouples:
Frequency Response

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ABSTRACT

Experimental measurements are made with a novel two wire thermocouple. Signals from two wires of unequal diameters are recorded from the thermocouple suspended in constant flow with a periodic temperature fluctuation. It is demonstrated that the reconstructed signal from the two wire thermocouple requires no compensation for $\omega \leq 2\,\omega_1$ where ω_1 is the natural frequency of the smaller wire. A compensation factor is recommended for larger frequencies $\omega > 2\omega_1$.

Theory and experimental measurements are compared with a novel three wire thermocouple. Signals from three wires of unequal diameters are recorded from the thermocouple suspended in constant flow with a periodic temperature fluctuation. It is demonstrated that the reconstructed signal from the three wire thermocouple requires no compensation for $\omega \leq 5\omega_1$ where ω_1 is the natural frequency of the smaller wire. The latter result represents a significant improvement compared to previous work with two wire thermocouples. A correction factor has also been derived to account for wires of arbitrary diameter.

Measurements are recorded for multiwire thermocouples consisting of either two or three wires of unequal diameters. Signals from the multiwire probe are recorded for a reversing gas flow with a periodic temperature fluctuation. It is demonstrated that the reconstructed signal from the multiwire thermocouple requires no compensation provided $\omega/\omega_1 < 2.3$ for two wires or $\omega/\omega_1 < 3.6$ for three wires where ω_1 is the natural frequency of the smaller wire based on the maximum gas velocity. The latter results were possible provided Fourier transformed data were used and knowledge of the gas velocity is available.

PREFACE

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I. TWO WIRE THERMOCOUPLE: FREQUENCY RESPONSE IN CONSTANT FLOW

1. INTRODUCTION

The evaluation of jet engine performance and fundamental studies of combustion phenomena depend on the measurement of turbulent fluctuating temperatures of the gas within the engine. Historically, these temperatures have been measured with thermocouples.[1] The advantages of thermocouples are their low cost, reliability, and simplicity since they do not require optical access or elaborate support electronics. However, the design of a thermocouple represents a compromise between accuracy, ruggedness, and rapidity of response.

For example, the measurement of fluctuating temperatures in the high-speed exhaust of a gas turbine engine combustor is required to characterize the local gas density gradients or convective heat transfer.[2] Although thermocouples are suitable for the measurement of high-frequency temperature fluctuations (<1 kHz) in a flowing gas or liquid, the measured signal must be compensated since the frequency of the time-dependent fluid temperature is normally much higher than the natural frequency of the thermocouple probe.[3] Moreover, use of a single wire thermocouple in constant velocity flows requires knowledge of the fluid velocity and properties (e.g., viscosity, density, etc.) to determine the natural frequency.

The present paper describes the performance of a novel two wire thermocouple of unequal diameters that does not require compensation at lower fluid temperature frequencies nor any knowledge of the fluid velocity or properties. The results of experimental measurements are presented along with the suggested procedure for the reduction of the data from the two wire thermocouple as shown in fig. 1.

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2. ONE WIRE

Use of a single wire thermocouple requires knowledge of the fluid velocity and properties to determine the natural frequency. The latter quantity is necessary to establish a frequency dependent compensation factor for the measured signal.

2.1 Steady State Response

The conservation of energy for a single wire is :

$$\frac{dT}{dt} = \omega_n (T_g - T) \tag{1}$$

where the natural frequency

$$\omega_n = \frac{4h}{\rho c_p D}. \tag{2}$$

Here:

ω_n	=	natural frequency (sec^{-1})
T	=	temperature of the thermocouple ($^\circ$K)
T_g	=	temperature of the gas ($^\circ$K)
t	=	time (sec)
ρ	=	wire density (kg/m^3)
c_p	=	heat capacity of wire (J/Kg - $^\circ$K)
h	=	heat transfer coefficient (W/m^2 - $^\circ$K)
D	=	wire diameter (m)

It is convenient to rewrite the heat transfer coefficient in terms of the Nusselt number

$$h = \frac{k_g}{D} Nu \tag{3}$$

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where

k_g = thermal conductivity of the gas (W/m - °K)

Nu = Nusselt number.

Thus, the natural frequency in Eq. (2) becomes

$$\omega_n = \frac{4k_g Nu}{(\rho c)D^2},$$ (4)

where $$Nu = C_o Re^m Pr^{1/3}.$$ (5)

and $$Re = \frac{VD}{\upsilon_g}$$

$$Pr = \frac{\mu c_p}{k_g}$$

We now assume a periodic variation in the temperature of the form

$$T_g = e^{i\omega t}$$ (6)

and a wire temperature

$$T = Ae^{i\omega t}.$$ (7)

Substituting Eqs. (6) and (7) into Eq. (1), one obtains a wire temperature or first order response to a simple harmonic gas temperature fluctuation of the form

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$$T = \frac{e^{i(\omega t + \phi_n)}}{\left[\left(\dfrac{\omega}{\omega_n}\right)^2 + 1\right]^{1/2}} \tag{8}$$

where the phase angle

$$\phi_n = -\tan^{-1}(\omega/\omega_n). \tag{9}$$

2.2 Error

The natural frequency of a single wire from Eqs. (4) and (5) can be written in the form,

$$\omega_n = \frac{k_g C_o 4 Re^m Pr^{1/3}}{(\rho c)D^2} \tag{10}$$

or separating the wire diameter

$$\omega_n = CD^{m-2} \tag{11}$$

where the coefficient C depends on both fluid and wire parameters or

$$C = \frac{4k_g C_o V^m Pr^{1/3}}{(\rho c)\upsilon_g^m}. \tag{12}$$

Substituting values corresponding to a type K thermocouple and a gas velocity $V = 10$ m/s for air at standard conditions into Eqs. (11) and (12), one obtains values of the natural frequency:

$\omega_n (sec^{-1})$	D (m)
33.8	50×10^{-6}
18.3	75×10^{-6}

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The values of the natural frequency ω_n depends on the choice of the exponent m in the empirical expressions Eqs. (4) and (5). Since a variation can exist in the value of m [4], a computation has been made to determine the effect of the magnitude of m on the amplitude of the steady state response. Thus, from Eq. (8) one computes an amplitude ratio defined as

$$\frac{T(m)}{T(.5)} = \left[\frac{\left(\dfrac{\omega}{\omega_n(.5)} \right)^2 + 1}{\left(\dfrac{\omega}{\omega_n(m)} \right)^2 + 1} \right]^{1/2} \tag{13}$$

where $\omega_n(m)$ is determined from Eqs. (11) and (12) for a wire diameter $D = 50 \times 10^{-6}$ m (50μm).

Figure 2 is a plot of Eq. (13) where it is demonstrated that a significant error can occur in the computation of the amplitude. In particular, an error of 40% in the amplitude will occur with a change of 20% in the magnitude of the exponent m. The problem can be significant for angular frequencies $\omega > \omega_n$

3. TWO WIRE

The use of a single wire thermocouple requires knowledge of the fluid velocity and additional properties such as viscosity or density. There are three unknowns that appear in Eqs. (1) and (11): a) the coefficient C given by Eq. (12) b) m the exponent of the wire Reynolds number in Eqs. (10), (11) and (12) and c) the gas temperature T_g.

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3.1 Determination of Gas Temperature

Consider two thermocouples of unequal diameters $D_1 < D_2$ as shown in fig. 1. The conservation of energy equation for both reduce to

$$\frac{dT_1}{dt} = CD_1^{m-2} (T_g - T_1)$$

(14)

$$\frac{dT_2}{dt} = CD_2^{m-2} (T_g - T_2).$$

(15)

Eliminating C and solving for T_g, one obtains

$$T_g = \frac{T_2\left(\dfrac{\dot{T}_1}{\dot{T}_2}\right) - T_1\left(\dfrac{D_1}{D_2}\right)^{m-2}}{\dfrac{\dot{T}_1}{\dot{T}_2} - \left(\dfrac{D_1}{D_2}\right)^{m-2}}$$

(16)

where T_1, T_2 are the measured temperatures and their derivatives from wires of diameter D_1, D_2, respectively. Use of Eq. (16) only requires an estimate of the exponent m and knowledge of the wire diameters D_1 and D_2.

The construction of the gas temperature from Eq. (16) also requires knowledge of the derivatives of the wire temperatures. Calculations of these derivatives may amplify noise in the sampled data. Since Eqs. (14) and (15) are linear equations, we choose to use transformed data profiles. Taking the Fast Fourier Transform (FFT) of Eqs. (14) and (15), one obtains

$$i\omega\overline{T_1} = CD_1^{m-2}(\overline{T}_g - \overline{T}_1)$$

(17)

$$i\omega\overline{T_2} = CD_2^{m-2}(\overline{T}_g - \overline{T}_2)$$

(18)

or solving for $\overline{T}_g$

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$$\overline{T}_g = \overline{T}_1 \left[\frac{1 - \left(\dfrac{D_1}{D_2}\right)^{m-2}}{\left(\dfrac{\overline{T}_1}{\overline{T}_2}\right) - \left(\dfrac{D_1}{D_2}\right)^{m-2}} \right].$$ (19)

The inverse transform of $\overline{T}_g$ is the reconstructed gas temperature or

$$T_g = FFT^{-1}\left[\overline{T}_g\right].$$ (20)

3.2 Error

Assuming the wire diameters are known accurately, the choice of the exponent m will affect the accuracy of the prediction of the gas temperature T_g as shown in Eq. (16). The effect of the exponent m on the predicted amplitude of T_g is shown in fig. 3 which plots the amplitude ratio $T_g(m)/T_g(.5)$ as a function of the normalized angular frequency ω/ω_1. In fig. 3 values of $D_1 = 50\mu m$ and $D_2 = 75\mu m$ were chosen including a value of the exponent m = 0.5. Substitution into Eq. (8) provided values of $T_1(D_1 = 50\mu)$ and $T_2(D_2 = 75\mu m)$ in Eq. (16).

As indicated in fig. 3, improper choice of the exponent m will create significant error in the prediction of the gas temperature. The error is magnified at larger frequencies $\omega \geq 3\,\omega_1$ for $\sim 25\%$ error in m while the onset of the error occurs at $\omega \geq 10\,\omega_1$ for a smaller $\sim 10\%$ error in m. Compared with similar computations for a single wire thermocouple in fig. 2: a) the error is much smaller b) the error for the two wire probe is shifted to larger frequencies $\omega > \omega_1$ and c) the error is reasonably constant for all choices of the exponent m.

Similar computations were made for a range of wire diameter ratios $1/2 \leq D_1/D_2 \leq 9/10$ with $D_1 = 50\mu m$. As shown in fig. 4, the error decreased by $\sim 12\%$ with wire diameters nearly equal.

4. EXPERIMENT

4.1 Apparatus

In the present experiment, thermocouple sensors are exposed to a constant velocity air stream (<19 m/s) of varying temperature. In particular, the dynamic response of the thermocouple is measured for a periodic temperature profile of varying frequency. A rotating wheel configuration is used to deliver the test air stream to the proposed sensors. A similar experimental apparatus was described in detail by Elmore et al[5] and Forney et al.[6]

A schematic of the rotating wheel apparatus used in the present experiment is shown in Fig. 5. As the wheel rotates, holes pass the two air supply tubes (3/4-in. i.d. copper) that allow slugs of hot (~55°C) and cold (~30°C) air to alternately enter a transition tube assembly mounted directly above the rotating wheel. In the transition tube the slugs of hot and cold air coalesce into a single air stream providing a periodic temperature profile covering a range of gas temperature frequencies from roughly 1 to 30 Hz. A 3/4-1/2 in. smooth copper adapter was inserted at the end of the coalescing tube (location of thermocouple) in Fig. 5 to increase the air velocity to 18 m/s.

The analog temperature signal is digitized with a Data Translation DT-2801 A/D board mounted in an expansion slot of an IBM AT compatible computer as shown in Fig. 5. ASYST software was loaded onto the hard disk of the personal computer and this provided a flexible system for data storage, manipulation, and display. The true temperature profile of the airstream is measured with a constant current anemometer (TSI 1054-A) and sensor (1210-T1.5).

The two type K thermocouple wires were of diameter $D_1 = 50\mu m$ and $D_2 = 75\mu m$ as shown in fig. 1. The thermocouple junctions were fabricated by the Research Instrumentation Branch at the NASA Lewis Center. The wires were cut

with a razor blade to produce a flat edge perpendicular to the wire axis. The wire segments were then mounted in a fixture such that the two faces of the junction were held together by springs. The junction was then formed by laser heating. The laser used was a KORAD model KWD Nd: YAG laser, operating at a wave length of 1060 nm. The power settings depend on the wire size and material, but for the thermocouples described in this report, a pulse duration of 4 ms was used delivering a total energy of approximately 2J.

4.2 Procedure

To calibrate the two thermocouples and anemometer, the air velocity at the exit of the coalescing tube in Fig. 5 (location of thermocouple) was maintained at roughly 17.5-18 m/s. The voltage regulator was adjusted such that the temperature of the hot line was maintained at an elevated temperature of ~55°C. One hole of the rotating wheel was positioned in a stationary position over the unheated line. The voltage output from both the thermocouple and constant current anemometer were recorded at the input terminals to the A/D board. The true temperature was also recorded with a thermometer suspended in the high-velocity air stream. This procedure was repeated for several temperatures with the hole positioned over the hot line covering an air temperature range from 30 to 55°C. It was found that both type K thermocouples had a constant calibration factor (°C/V) over the small temperature range. The constant current anemometer, however, provided a nonlinear response with the calibration factor varying with air temperature. Thus, a least squares curve fit was determined to provide the gas temperature with the anemometer output. The calibration factors described above were then installed on the ASYST data acquisition software.

The thermocouple and constant current anemometer are mounted in the constant velocity air stream with both sensing elements parallel and separated by

approximately 1 mm. The period or frequency of the periodic temperature profile is controlled over the range from 1.5 to 30 Hz with adjustment of the motor connected to the rotating wheel shaft.

ASYST software was developed to acquire temperature data sequentially from the thermocouples and constant current anemometer. The data are digitized for three channels at a sampling rate of 1024 Hz per channel for a total sample time of 1 s. After data acquisition, the Fast Fourier Transform (FFT) is taken for each channel. The largest peak (first harmonic) in the amplitude of the FFT is located for each channel which provides the frequency of the temperature profile. The ASYST software also records the amplitude ratio of the first harmonic for the 50μm thermocouple - to - anemometer output. The amplitude ratio was recorded for each frequency setting of the rotating wheel and provided a value for the natural frequency ω_1 of the small wire.

The FFT from each thermocouple channel was substituted into Eq. (19) along with values for the diameter ratios and an estimate of the exponent m by the ASYST code. The inverse transform of $\overline{T}_g$ by the ASYST code provided a reconstruction of the true gas temperature.

5. RESULTS AND DISCUSSION

Temperature profiles from the two thermocouples and anemometer were recorded for several angular frequencies of the rotating wheel. Figure 6 illustrates three profiles covering a time span of 0.3 s for a wheel angular frequency of $\omega = 74$ s^{-1} and a natural frequency $\omega_1 = 42$ s^{-1} for the smaller 50μm wire. Here, the attenuation and phase shift are evident for the two interior profiles corresponding to the two wire thermocouple response.

The FFT of the two thermocouple signals was determined and the results were combined as suggested by Eq. (19). The inverse FFT was taken of $\overline{T}_g$ and the result was plotted along with the anemometer output as shown in fig. 7. As indicated, the anemometer output and the reconstructed signal from the two thermocouples are nearly equal.

A series of experiments were performed with the wheel frequency covering a range of values from 2.5 to 25 Hz. The amplitude ratio of the reconstructed signal to the gas temperature (anemometer output) was plotted versus ω/ω_1 as shown in fig. 8. Each data point in fig. 8 represents the average ratio from two consecutive peaks derived from plots similar to fig. 7.

For fixed exponent m = 0.4, fig. 8 illustrates that the reconstructed signal requires no compensation until $\omega \simeq 2\omega_1$. For larger frequencies $\omega > 2\omega_1$, the data conform to the empirical function for the amplitude ratio

$$\frac{\left|FFT^{-1}\left[\overline{T}_g\right]\right|}{|T_g|} = 1.36 - 0.265(\omega/\omega_1) \tag{21}$$

where the numerator is derived from Eqs. (19) and (20) and the denominator is the output amplitude from the anemometer. Also shown for comparison in fig 8 is the first order response for a single thermocouple wire.

It is possible to predict the natural frequency of the small wire ω_1 from Eqs. 11, 17 and 18. Eliminating the transform of the gas temperature $\overline{T}_g$ from Eqs. 17 and 18, one obtains

$$\omega_1 = i\omega\left[\frac{(D_1/D_2)^{m-2}\,\overline{\overline{T}}_2 - \overline{\overline{T}}_1}{(\overline{\overline{T}}_1 - \overline{\overline{T}}_2)}\right]. \tag{22}$$

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Equation (22) is a useful expression provided the first harmonic frequency from the transform of the gas temperature T_g is below $2\omega_1$. Evidence to support the latter must be derived from prior knowledge and estimates of the gas velocities, properties and the temperature fluctuation frequency. Knowledge of the natural frequency ω_1 from Eq. (22) would allow one to use the compensation factor Eq. (21) for the amplitudes of the higher harmonics of T_g where $\omega > 2\omega_1$.

From Eq. (19), the amplitude ratio should depend on the exponent m. Additional data were recorded for $m = 0.3, 0.4, 0.5$ and 0.6 and the results are shown in fig. 9. It is apparent that m has little influence on the amplitude ratio. Theoretical results suggest that there must be a single value of m that would provide an amplitude ratio of one for all frequencies. However, the empirical nature of Eq. (5) for the Nusselt number and sampling error provide the attenuation in Fig. 8.

6. CONCLUSIONS

Experimental measurements are made with a novel two wire thermocouple. Signals from two wires of unequal diameters are recorded from the thermocouple suspended in constant flow with a periodic temperature fluctuation. It is demonstrated that the reconstructed signal from the two wire thermocouple requires no compensation for $\omega \leq 2\,\omega_1$ where ω_1 is the natural frequency of the smaller wire. A compensation factor is recommended for larger frequencies $\omega > 2\omega_1$.

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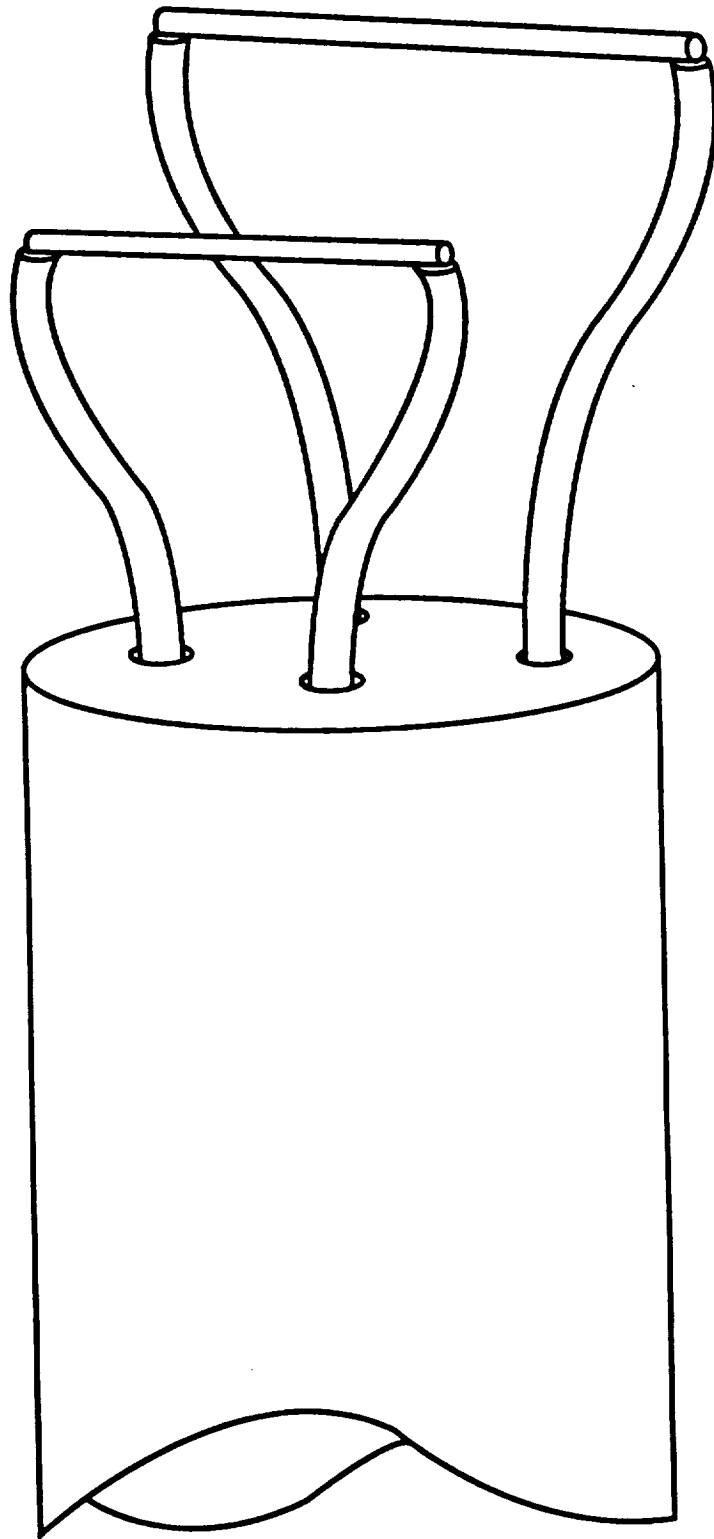
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8. FIGURES

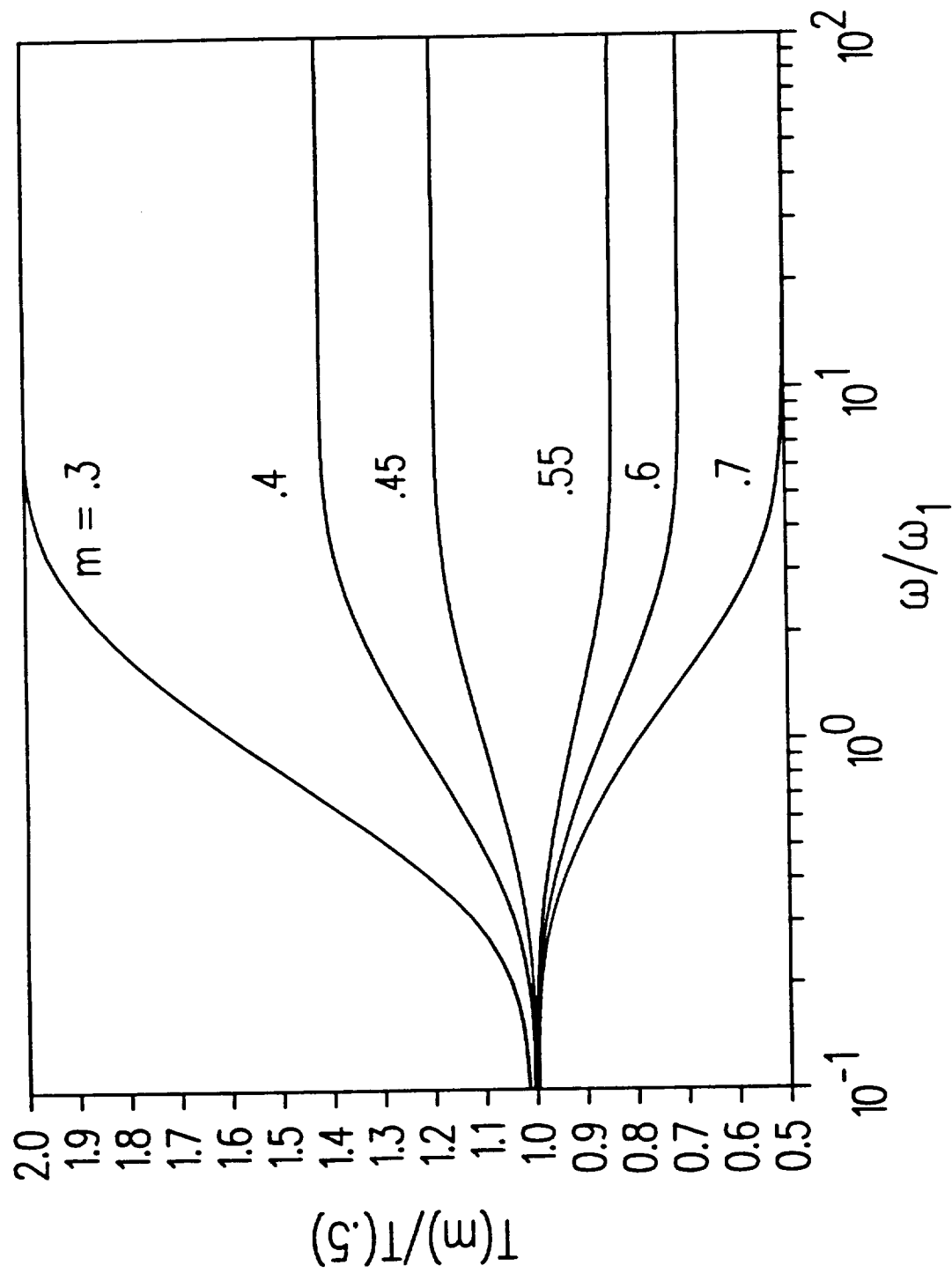Fig. 1 -   Schematic of two wire thermocouple.

Fig. 2 - Effect of exponent m on the amplitude for single wire.

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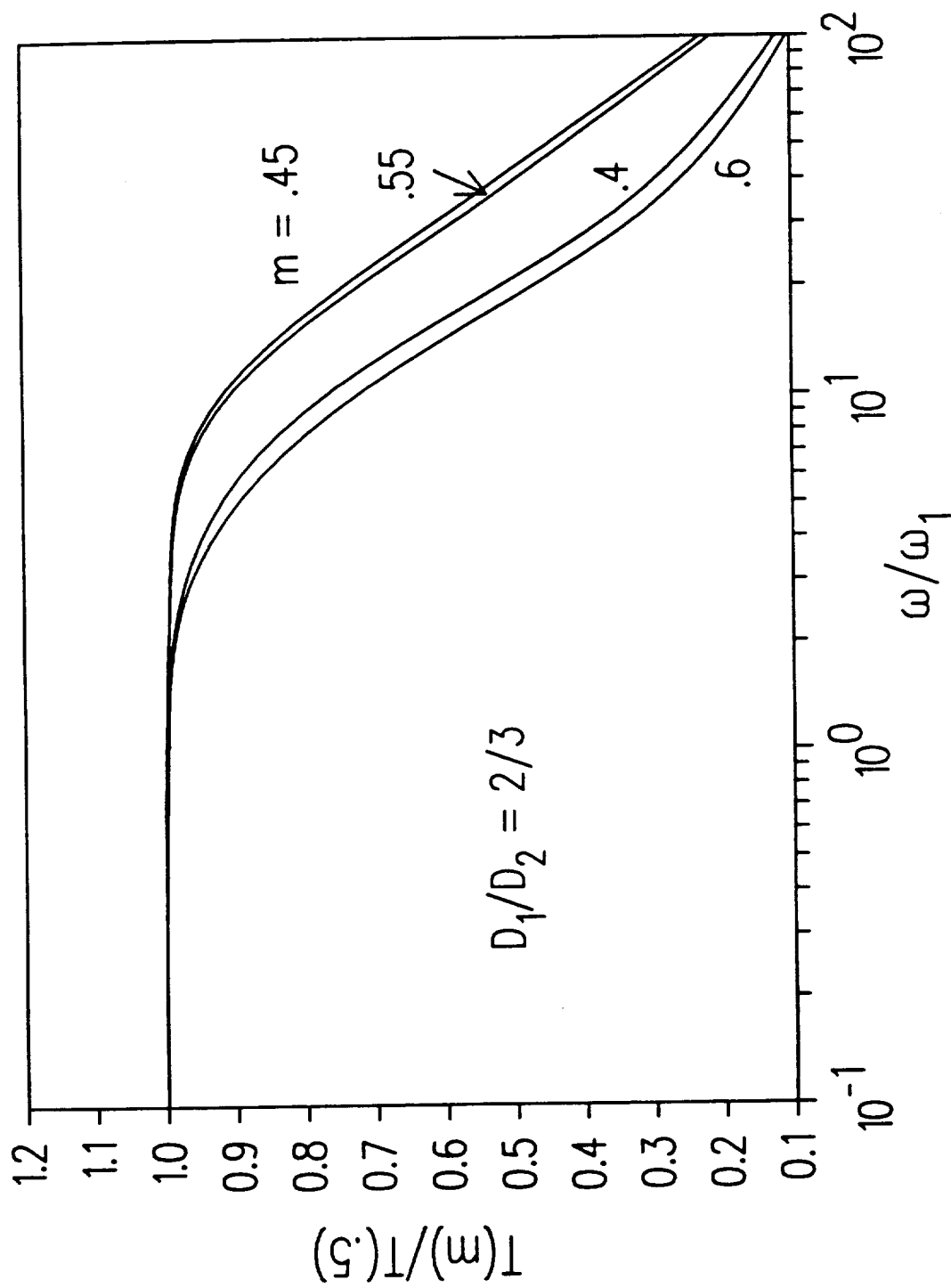
Fig. 3 - Effect of exponent m on the amplitude for a two wire thermocouple.

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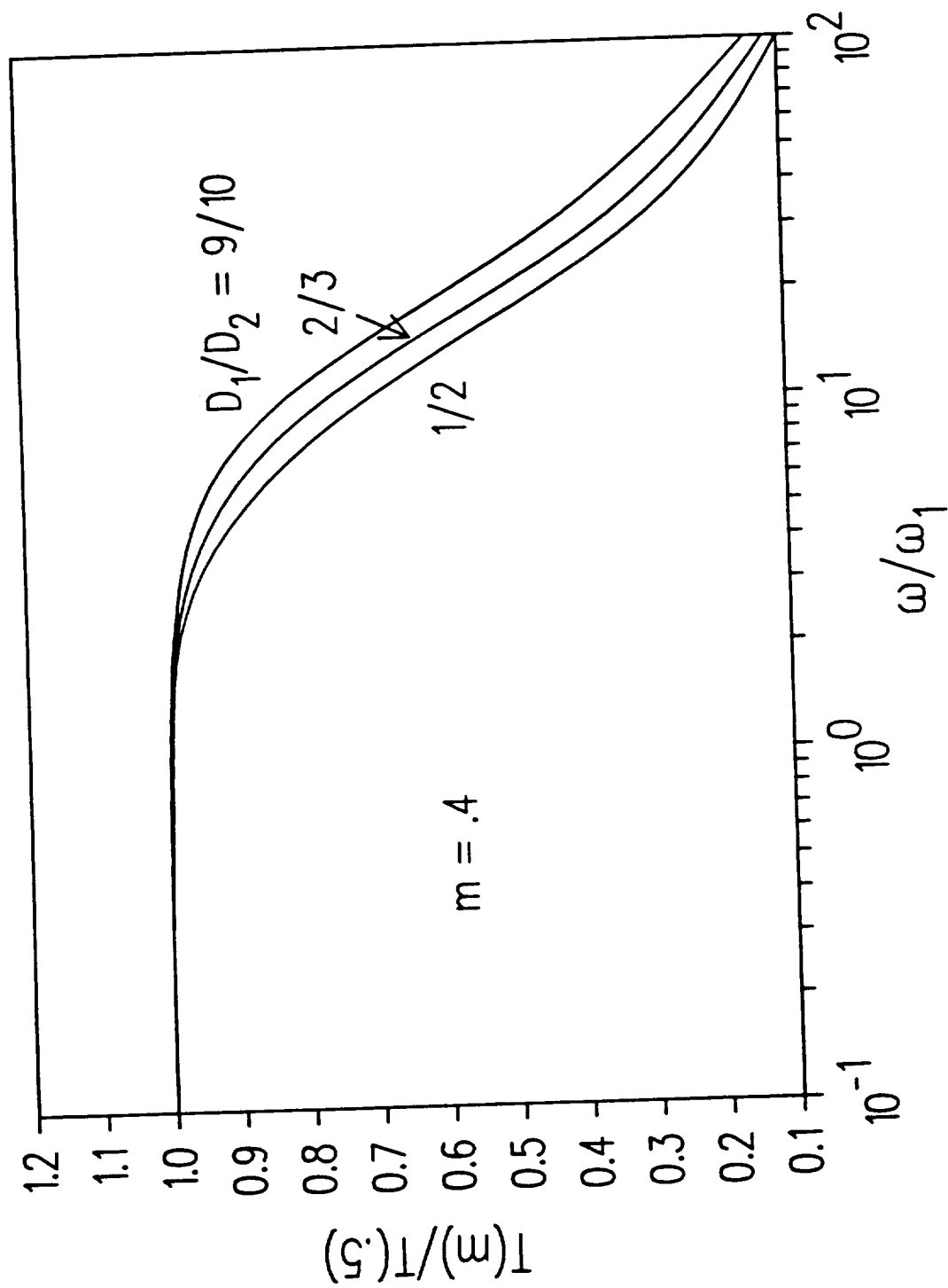
Fig. 4 -   Effect of wire diameter ratio on the amplitude.

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HIGH PRESSURE
AIR

PRESSURE
REGULATOR

THERMOCOUPLE

CONSTANT CURRENT
ANEMOMETER

ICE
BATH

HEATING
TAPE

HIGH VELOCITY
AIRSTREAM

TSI 1054-A
ANEMOMETER

AMPLIFIER

HOT   COLD

ROTATING
WHEEL

DT-2800
A/D BOARD

MOTOR

INSULATION

PERSONAL
COMPUTER
W/ ASYST SOFTWARE

PRINTER

STORAGE
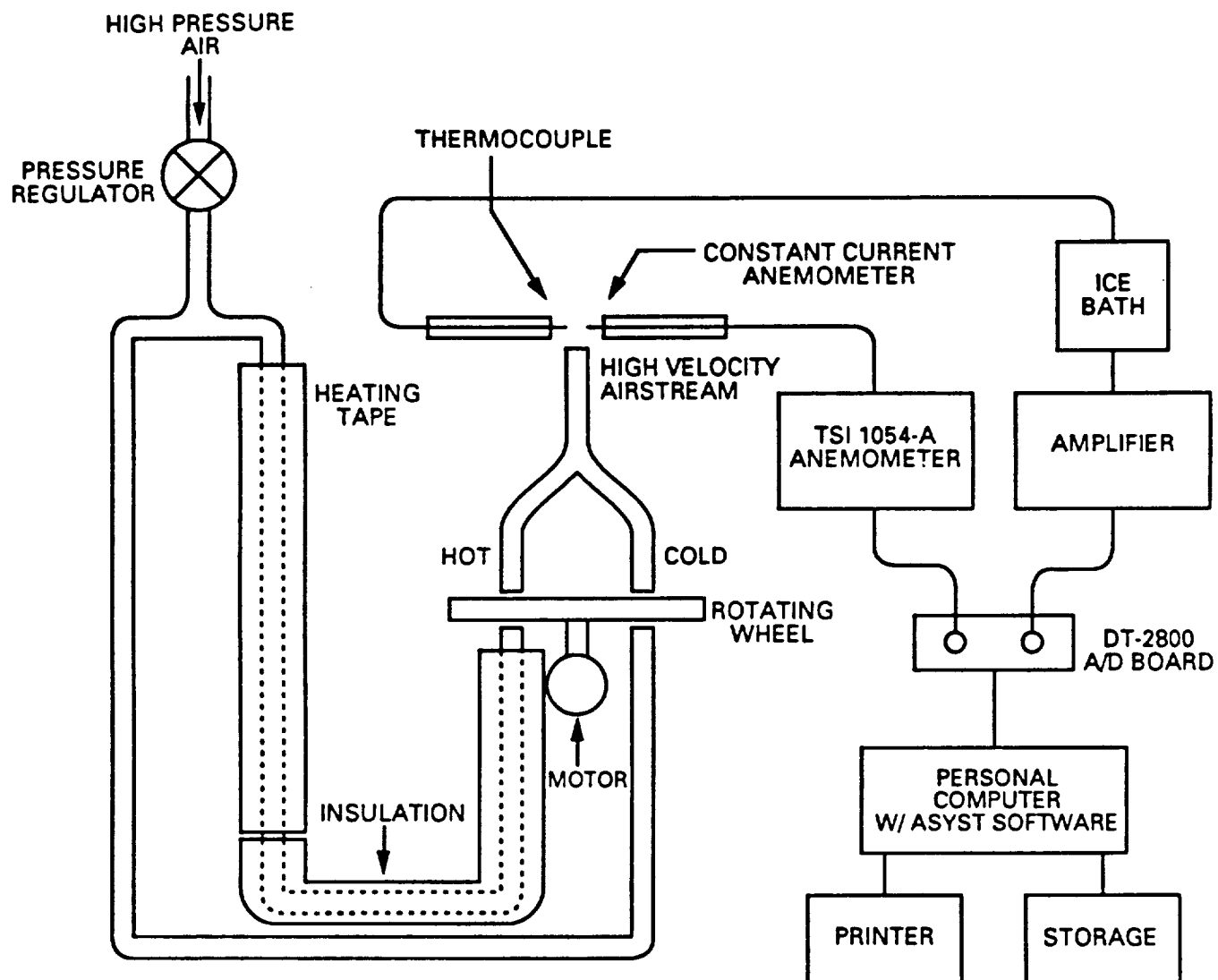
Fig. 5 -   Schematic of rotating wheel apparatus.
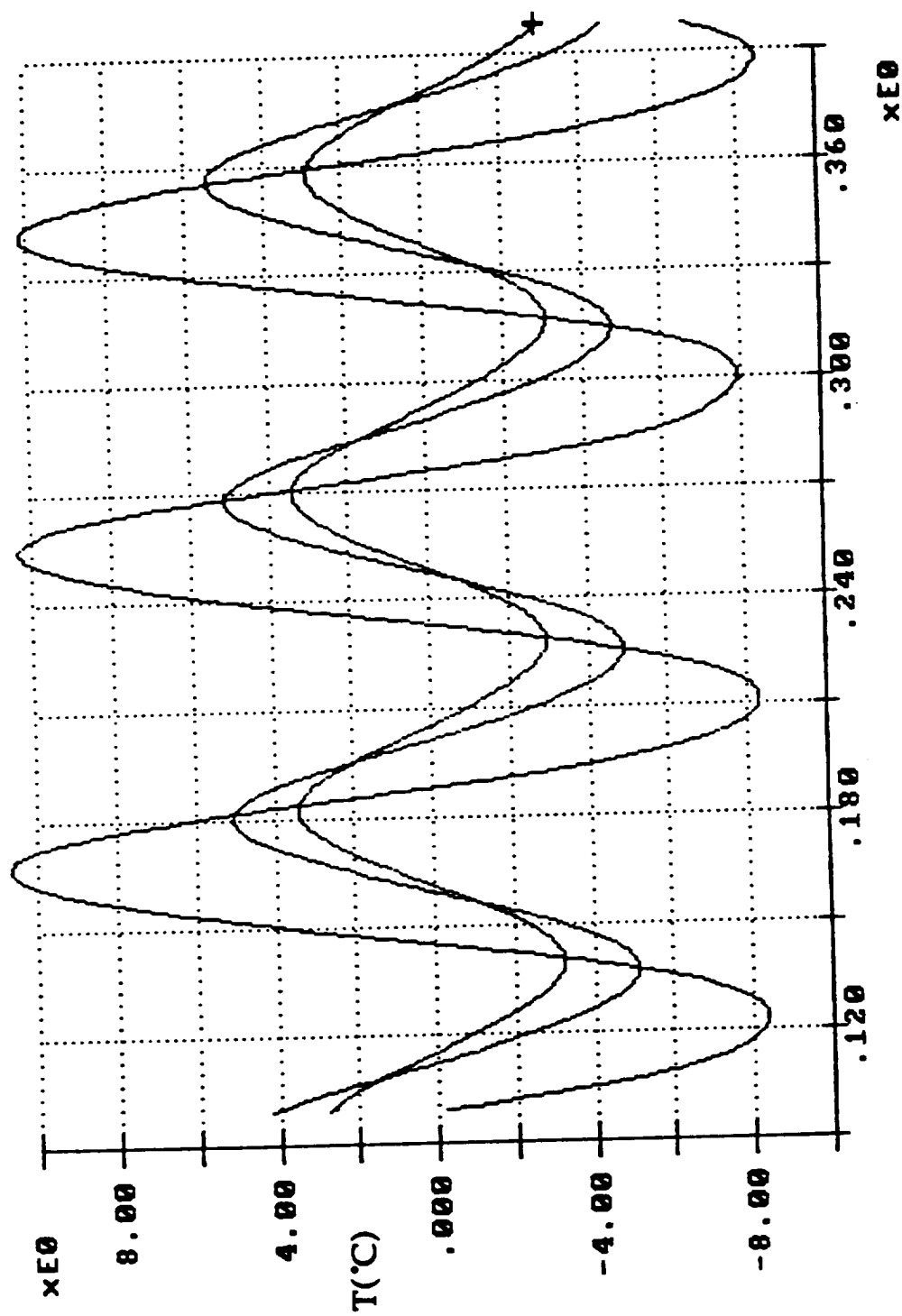
Fig. 6 - Temperature profiles for the gas (large amplitude) and two type K thermocouple wires. Two wires: $D_1 = 50\mu m$, $\omega_1 = 42\ s^{-1}$, $D_2 = 75\mu m$. Gas temperature frequency: $\omega = 74\ s^{-1}$.

Fig. 7 - Gas temperature profile ($\omega = 74$ s^{-1}) compared to reconstructed temperature derived from two wire thermocouple.

Fig. 8 - Amplitude ratio of two wire thermocouple output-to-true gas temperature.

Fig. 9 -   Effect of exponent m on the amplitude ratio.

II. TWO WIRE THERMOCOUPLE: FREQUENCY RESPONSE IN CONSTANT FLOW

1. INTRODUCTION

The evaluation of jet engine performance and fundamental studies of combustion phenomena depend on the measurement of turbulent fluc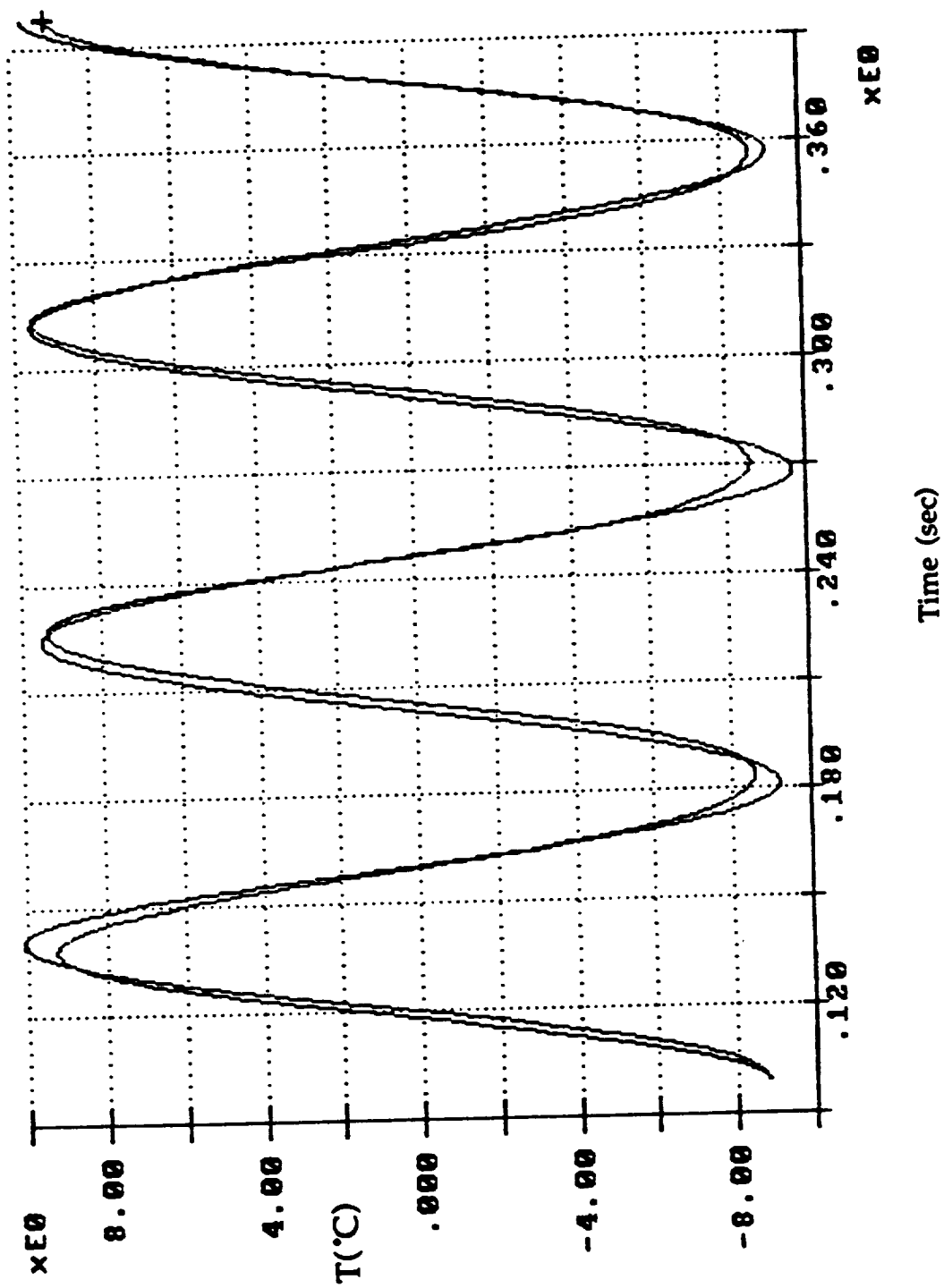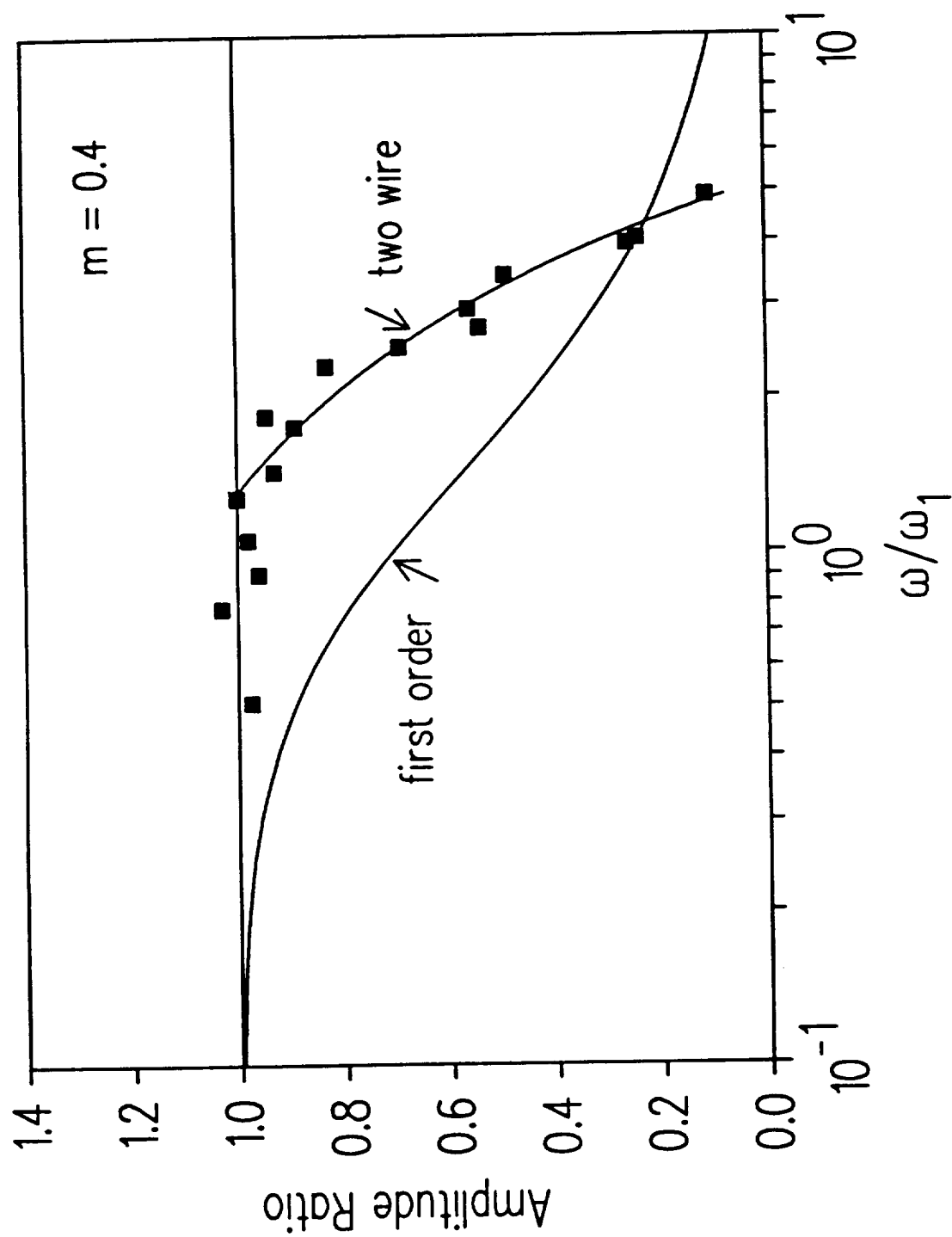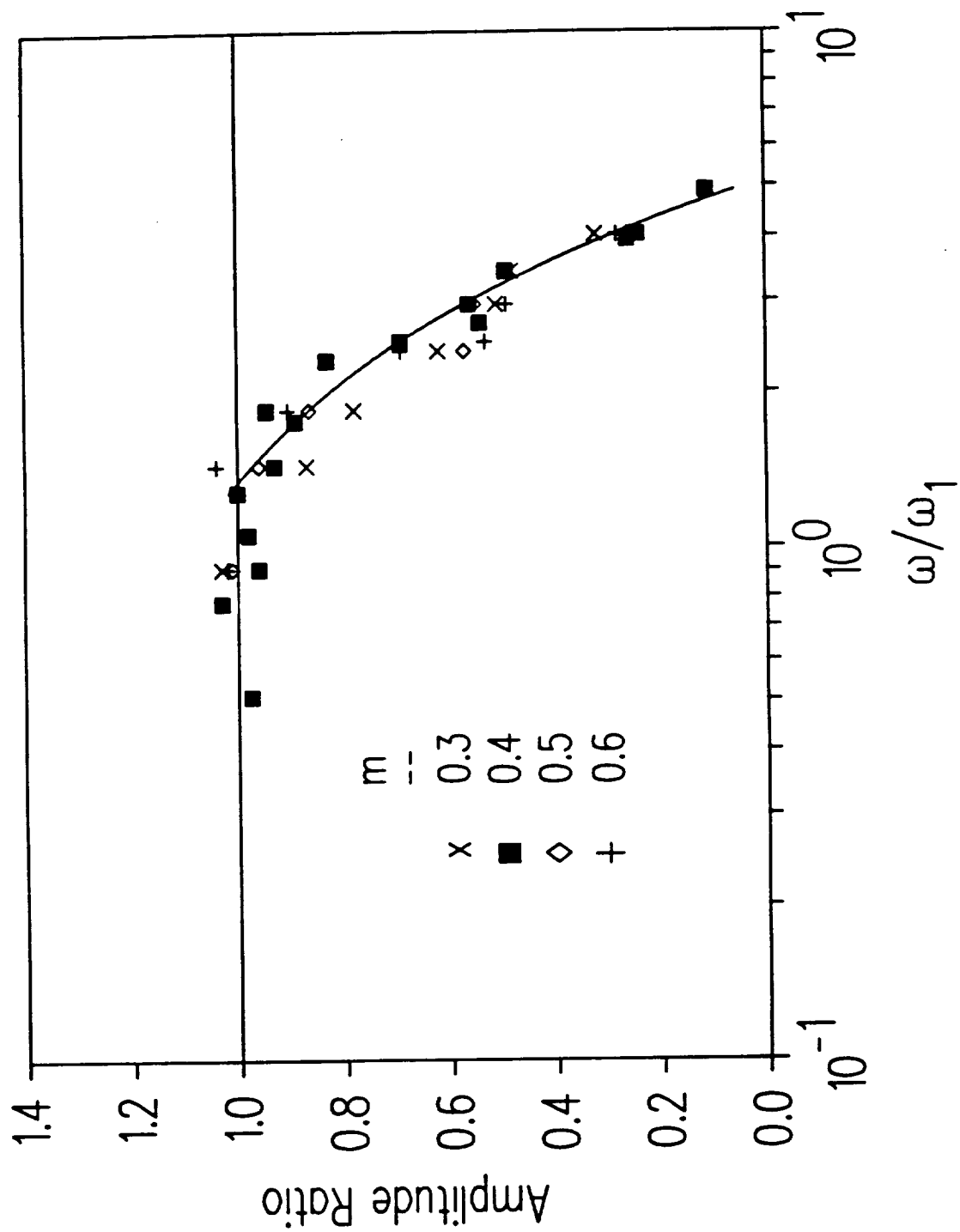tuating temperatures of the gas within the engine. Historically, these temperatures have been measured with thermocouples.[1] The advantages of thermocouples are their low cost, reliability, and simplicity since they do not require optical access or elaborate support electronics. However, the design of a thermocouple represents a compromise between accuracy, ruggedness, and rapidity of response.

For example, the measurement of fluctuating temperatures in the high-speed exhaust of a gas turbine engine combustor is required to characterize the local gas density gradients or convective heat transfer.[2] Although thermocouples are suitable for the measurement of high-frequency temperature fluctuations (<1 kHz) in a flowing gas or liquid, the measured signal must be compensated since the frequency of the time-dependent fluid temperature is normally much higher than the natural frequency of the thermocouple probe.[3] Moreover, use of a single wire thermocouple in constant velocity flows requires knowledge of the fluid velocity and properties (e.g., viscosity, density, etc.) to determine the natural frequency.

The present paper describes the performance of a novel three wire thermocouple of unequal diameters that does not require compensation at lower fluid temperature frequencies nor any knowledge of the fluid velocity or properties.[4,5] The results of experimental measurements are presented along with the suggested procedure for the reduction of the data from the three wire thermocouple as shown in fig. 1.

2. THEORY

Use of a single wire thermocouple requires knowledge of the fluid velocity and properties to determine the natural frequency. The latter quantity is necessary to establish a frequency dependent compensation factor for the measured signal.

2.1 Natural Frequency of Single Wire

The conservation of energy for a single wire is :

$$\frac{dT}{dt} = \omega_n (T_g - T) \tag{1}$$

where the natural frequency

$$\omega_n = \frac{4h}{\rho c D}. \tag{2}$$

Here:

ω_n	=	natural frequency (sec^{-1})
T	=	temperature of the thermocouple (°K)
T_g	=	temperature of the gas (°K)
t	=	time (sec)
ρ	=	wire density (kg/m^3)
c	=	heat capacity of wire (J/kg - °K)
h	=	heat transfer coefficient (W/m^2 - °K)
D	=	wire diameter (m)

It is convenient to rewrite the heat transfer coefficient in terms of the Nusselt number

$$h = \frac{k_g}{D} Nu \tag{3}$$

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where

$$k_g \quad = \quad \text{thermal conductivity of the gas (W/m - °K)}$$

$$Nu \quad = \quad \text{Nusselt number.}$$

Thus, the natural frequency in Eq. (2) becomes

$$\omega_n \;=\; \frac{4k_g Nu}{(\rho c)D^2}, \tag{4}$$

where[6]
$$Nu \;=\; C_o\, Re^m\, Pr^{1/3}. \tag{5}$$

and
$$Re \;=\; \frac{VD}{\upsilon_g}$$

$$Pr \;=\; \frac{\mu c_p}{k_g}$$

Here, Re and Pr are the Reynolds and Prandtl numbers, respectively and

$$\mu \quad = \quad \text{viscosity of gas (kg/m-sec)}$$

$$\upsilon_g \quad = \quad \text{Kinematic viscosity of gas (m}^2\text{ - sec}^{-1}\text{).}$$

$$V \quad = \quad \text{velocity of gas (m/s)}$$

The natural frequency of a single wire from Eqs. (4) and (5) can now be written in the form,

$$\omega_n \;=\; \frac{k_g C_o\, 4\, Re^m\, Pr^{1/3}}{(\rho c)D^2} \tag{6}$$

or separating the wire diameter

$$\omega_n = CD^{m-2} \tag{7}$$

where the coefficient C depends on both fluid and wire parameters or

$$C = \frac{4k_g C_o V^m Pr^{1/3}}{(\rho c)\upsilon_g^m}. \tag{8}$$

Substituting values corresponding to a type K thermocouple and a gas velocity $V = 25$ m/s for air at standard conditions into Eqs. (7) and (8), one obtains values of the natural frequency:

$\omega_n(\text{sec}^{-1})$	D (m)	D(mil)
52.6	50.8×10^{-6}	2
18.3	101.6×10^{-6}	4
6.5	203.2×10^{-6}	8

2.2 Determination of Gas Temperature

The use of a single wire thermocouple requires knowledge of the fluid velocity and additional properties such as viscosity or density. There are three unknowns that appear in Eqs. (1) and (7): a) the coefficient C given by Eq. (8) b) m the exponent of the wire Reynolds number in Eqs. (6), (7) and (8) and c) the gas temperature T_g.

Consider three thermocouples of unequal diameters $D_1 < D_2 < D_3$ as shown in fig. 1. The conservation of energy equation for each wire reduces to

$$\frac{dT_1}{dt} = CD_1^{m-2} (T_g - T_1) \tag{9}$$

$$\frac{dT_2}{dt} = CD_2^{m-2} (T_g - T_2). \tag{10}$$

$$\frac{dT_3}{dt} = CD_3^{m-2}(T_g - T_3). \tag{11}$$

In principle it is possible to solve for the three unknowns that appear in Eqs. (9)-(11) in terms of the instantaneous values of the wire temperatures T_1, T_2 and T_3 and their derivatives. However, because of the empirical nature of the Nusselt number given in Eq. (5) and uncertainty in the wire diameters near the thermocouple junctions, we choose to eliminate the coefficient C and solve for the gas temperature Tg from Eqs. (9)-(11). Moreover, because the conservation of energy equations are linear at constant ambient velocities, we choose to use transformed data profiles.

Thus, taking the Fast Fourier Transform (FFT) of Eqs. (9)-(11), squaring Eq. (10) and dividing by the product of Eqs (9) and (11), one obtains

$$\left(\frac{\overline{T}_2^2}{\overline{T_1 T_3}}\right) = \left(\frac{D_2^2}{D_1 D_3}\right)^{m-2} \frac{(\overline{T}_g - \overline{T}_2)^2}{(\overline{T}_g - \overline{T}_1)(\overline{T}_g - \overline{T}_3)}. \tag{12}$$

Substituting $\qquad r = \overline{T}_2^2 / \overline{T}_3 \overline{T}_1 \tag{13}$

into Eq. (12) and anticipating that the wire diameter ratio $D_2^2 / D_1 D_3 \sim O(1)$ or

$$\left(\frac{D_2^2}{D_1 D_3}\right)^{m-2} = 1 + \epsilon, \tag{14}$$

the transform of the gas temperature becomes

$$\overline{T}_g = \frac{-b_o - \sqrt{b_o^2 - 4a_o c_o}}{2a_o}. \tag{15}$$

Here,

$$b_o = b + 2\epsilon \overline{T}_2 + O(\epsilon)^2$$

$$-4a_o c_o = 4\epsilon \overline{T}_2^2 (r-1) + O(\epsilon)^2$$

$$b_o^2 - 4a_o c_o = b^2 + \epsilon \gamma + O(\epsilon)^2$$

and

$$a_o = r - 1 - \epsilon$$

where

$$b = 2\overline{T}_2 - r(\overline{T}_3 + \overline{T}_1) \tag{16}$$

and

$$\gamma = 4T_2(b + \overline{T}_2(r-1)).$$

We now seek a Taylor series expansion of Eq. (15) for the transformed gas temperature about the point $\epsilon = 0$. Thus, to first order in ϵ where

$$f(\epsilon) = \sqrt{b_o^2 - 4a_o c_o} = \sqrt{b^2 + \epsilon \gamma}$$

and

$$g(\epsilon) = \frac{1}{a_o} = \frac{1}{r - 1 - \epsilon},$$

one obtains

$$f(\epsilon) = b + \frac{\gamma}{2b}\epsilon + O(\epsilon)^2 \tag{17}$$

and

$$g(\epsilon) = \frac{1}{a} + \frac{\epsilon}{a^2} + O(\epsilon)^2 \tag{18}$$

where

$$a = r - 1. \tag{19}$$

Substituting Eqs. (17) and (18) into Eq. (15) and retaining terms $O(\epsilon)$, one obtains

$$\overline{T}_g = -\frac{b}{a} - \epsilon\left(\frac{2\overline{T}_2}{a} + \frac{\overline{T}_2^2}{b} + \frac{b}{a^2}\right) \tag{20}$$

where a, b are defined by Eqs. (13), (19) and (16), respectively.

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We now define the transform of the gas temperature for the particular case where the wire diameter ratio $D_2^2 / D_1 D_3 = 1$ in the form

$$\overline{T}_g(0) = -\frac{b}{a} \qquad (21)$$

or

$$\overline{T}_g(0) = \frac{\overline{T}_2(\overline{T}_2\overline{T}_1 + \overline{T}_2\overline{T}_3 - 2\overline{T}_1\overline{T}_3)}{(\overline{T}_2^2 - \overline{T}_1\overline{T}_3)}. \qquad (22)$$

Substituting for the quantity b defined by Eq. (21) into Eq. (20), one obtains the final result

$$\overline{T}_g(\epsilon) = \overline{T}_g(0) + \epsilon \left(\frac{1}{a\overline{T}_g(0)}\right)\left(\overline{T}_2 - \overline{T}_g(0)\right)^2 \qquad (23)$$

where

$$a = \frac{\overline{T}_2^2}{\overline{T}_1\overline{T}_3} - 1$$

and $\overline{T}_g(0)$ is defined by Eq. (22). The inverse transform of $\overline{T}_g(\epsilon)$ becomes the reconstructed gas temperature or

$$T_g = FFT^{-1}\left[\overline{T}_g(\epsilon)\right] \qquad (24)$$

for assumed values of ϵ.

It is useful to consider the effects of wire diameter on the magnitude of the perturbation parameter ϵ. In general it is desirable to have small diameter wires so that their natural frequencies are shifted to larger values. Listed in table 1 are three possible wire combinations that are available commercially along with the magnitude of ϵ. The values of ϵ are computed from Eq. (14) assuming that the wire diameters are exactly as listed by the manufacturer which is very unlikely in practice. For example, a typical value of the exponent m = 1/2 in Eq. (14) suggests

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that a 7% error in D_2 would yield a value of $\epsilon \lesssim 0.2$ for the wire diameters listed in the first row of Table 1.

TABLE 1 - Wire Diameter Combinations (m = 1/2)

D_1(mil)	D_2(mil)	D_3(mil)	ϵ
2	4	8	0.0
2	3	5	0.17
2	3	4	−0.16

3. EXPERIMENT

3.1 Apparatus

In the present experiment, thermocouple sensors are exposed to a constant velocity air stream (<25 m/s) of varying temperature. In particular, the dynamic response of the thermocouple is measured for a periodic temperature profile of varying frequency. A rotating wheel configuration is used to deliver the test air stream to the proposed sensors. A similar experimental apparatus was described in detail by Elmore et al[7] and Forney et al.[8]

A schematic of the rotating wheel apparatus used in the present experiment is shown in Fig. 2. As the wheel rotates, holes pass the two air supply tubes (3/4-in. i.d. copper) that allow slugs of hot (~55°C) and cold (~30°C) air to alternately enter a transition tube assembly mounted directly above the rotating wheel. In the transition tube the slugs of hot and cold air coalesce into a single air stream providing a periodic temperature profile covering a range of gas temperature frequencies from roughly 1 to 60 Hz. A 3/4-1/2 in. smooth copper adapter was inserted at the end of the coalescing tube (location of thermocouple) in Fig. 2 to increase the air velocity to 25 m/s.

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The analog temperature signal is digitized with a Data Translation DT-2801 A/D board mounted in an expansion slot of an IBM AT compatible computer as shown in Fig. 2. ASYST software was loaded onto the hard disk of the personal computer and this provided a flexible system for data storage, manipulation, and display. The true temperature profile of the airstream is measured with a constant current anemometer (TSI 1054-A) and sensor (1210-T1.5).

The three type K thermocouple wires were of diameter $D_1 = 50.8\mu m$ (2 mil), $D_2 = 101.6\mu m$ and $D_3 = 203.2\mu m$ as shown in fig. 1. The thermocouple junctions were fabricated by the Research Instrumentation Branch at the NASA Lewis Center. The wires were cut with a razor blade to produce a flat edge perpendicular to the wire axis. The wire segments were then mounted in a fixture such that the two faces of the junction were held together by springs. The junction was then formed by laser heating. The laser used was a KORAD model KWD Nd: YAG laser, operating at a wave length of 1060 nm. The power settings depend on the wire size and material, but for the thermocouples described in this report, a pulse duration of 4 ms was used delivering a total energy of approximately 2J.

3.2 Procedure

ASYST software was developed to acquire temperature data sequentially from the thermocouples and constant current anemometer. The data are digitized for four channels at a sampling rate of 512 Hz per channel for a total sample time of 0.5 s. After data acquisition, the Fast Fourier Transform (FFT) is taken for each channel. The largest peak (first harmonic) in the amplitude of the FFT is located for each channel which provides the frequency of the temperature profile. The ASYST software also records the amplitude ratio of the first harmonic for the 50.8μm thermocouple - to - anemometer output. The amplitude ratio was recorded for each

frequency setting of the rotating wheel and provided a value for the natural frequency ω_1 of the small wire.

The FFT of the signal from each thermocouple channel was substituted into Eq. (19) by the ASYST code along with an estimate of the parameter ϵ. The inverse transform of $\overline{T}_g$ by the ASYST code provided a reconstruction of the true gas temperature.

4. RESULTS AND DISCUSSION

The temperature profiles for the ambient gas (large amplitude) and the three type K thermocouple wires are shown in fig. 3. The three wire diameters used in the present study are $D_1 = 50.8\mu m$ (2mil), $D_2 = 101.6\mu m$ and $D_3 = 203.2\mu m$. The angular frequency of the gas temperature in fig. 3 is $\omega = 86$ s^{-1} (~ 14 Hz) while the natural frequency of the small wire $\omega_1 = 42$ s^{-1}. The natural frequency of the small wire corresponds to an ambient gas velocity of roughly 10 m/s.

Figures 4, 5 and 6 illustrate the effects of increasing the magnitude of the wire diameter parameter ϵ defined by Eq. (14). The small phase angle between the large ambient gas temperature profiles that appear in figs. 4 and 5 and the smaller temperature profiles reconstructed from the thermocouple probe allow one to distinguish between both profiles in fig. 6. In the latter case, the temperature profile on the left is that of the ambient gas while the profile on the right was determined by taking the inverse FFT of $\overline{T}_g(\epsilon)$ defined by Eq. (23) with $\epsilon = 0.2$. In all three figures the natural frequency of the small wire $\omega_1 = 61$ s^{-1} (V $\simeq$ 20 ms^{-1}) while the angular frequency of the ambient gas temperature is $\omega = 209$ s^{-1}(~ 33 Hz).

It is clear that a value of the parameter $\epsilon = 0.2$ provides a signal that requires no compensation for the frequency ratio $\omega/\omega_1 = 3.4$. It should also be noted that a 7% error in the diameter of D_2 would account for a value of $\epsilon = 0.2$. Further, the

coefficient of ϵ in Eq. (23) depends on the difference between the uncompensated FFT of the signal from the wire of diameter D_2 and the compensated FFT of the signal from the three wire probe with a particular diameter ratio $D_2^2 = D_1 D_3$ ($\epsilon = 0$). Thus, the contribution to the signal by the second term on the right of Eq. (23) is negligible at low frequencies but increases with larger values of ω/ω_1.

The amplitude ratio of the three wire thermocouple output-to-true gas temperature as a function of gas temperature frequency has been plotted in fig. 7. Also shown is the first order response of the small wire. The open symbols correspond to a value of $\epsilon = 0$ in Eq. (23) where the amplitude ratio requires no compensation for $\omega \leq 2\omega_1$. For the range of values $2\omega_1 < \omega < 5\omega_1$ the amplitude ratio follows the function

$$A = 1.92\left(\frac{\omega}{\omega_1}\right)^{-1.03}. \tag{25}$$

It should be noted that the first order response is also proportional to $(\omega/\omega_1)^{-1}$ at large frequencies.

A significant improvement is observed, however, when the parameter $\epsilon = 0.2$ as indicated by the solid symbols in fig. 7. For the latter case, no compensation is required for $\omega \leq 5\omega_1$. As discussed earlier, the amplitude ratio data coelesce at low gas temperature frequencies where the coefficient of ϵ in Eq. (23) approaches zero.

Both values of $\epsilon = 0.0$ or 0.2 are of little value, however, when $\omega \geq 6\omega_1$ where the amplitude ratio for the three wire thermocouple reduces to the first order response of the smaller wire. The latter result appears to occur at a gas frequency of ~ 60 Hz. For the present experimental system the signal - to - noise ratio may be responsible for the small response at large frequencies. This situation may be improved by replacing the two larger

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wires of 4 and 8 mil diameter by wires of 3 and 4 mils, respectively, as listed in table 1.

5. CONCLUSIONS

Theory and experimental measurements are compared with a novel three wire thermocouple. Signals from three wires of unequal diameters are recorded from the thermocouple suspended in constant flow with a periodic temperature fluctuation. It is demonstrated that the reconstructed signal from the three wire thermocouple requires no compensation for $\omega \leq 5\omega_1$ where ω_1 is the natural frequency of the smaller wire. The latter result represents a significant improvement compared to previous work with two wire thermocouples. A correction factor has also been derived to account for wires of arbitrary diameter.

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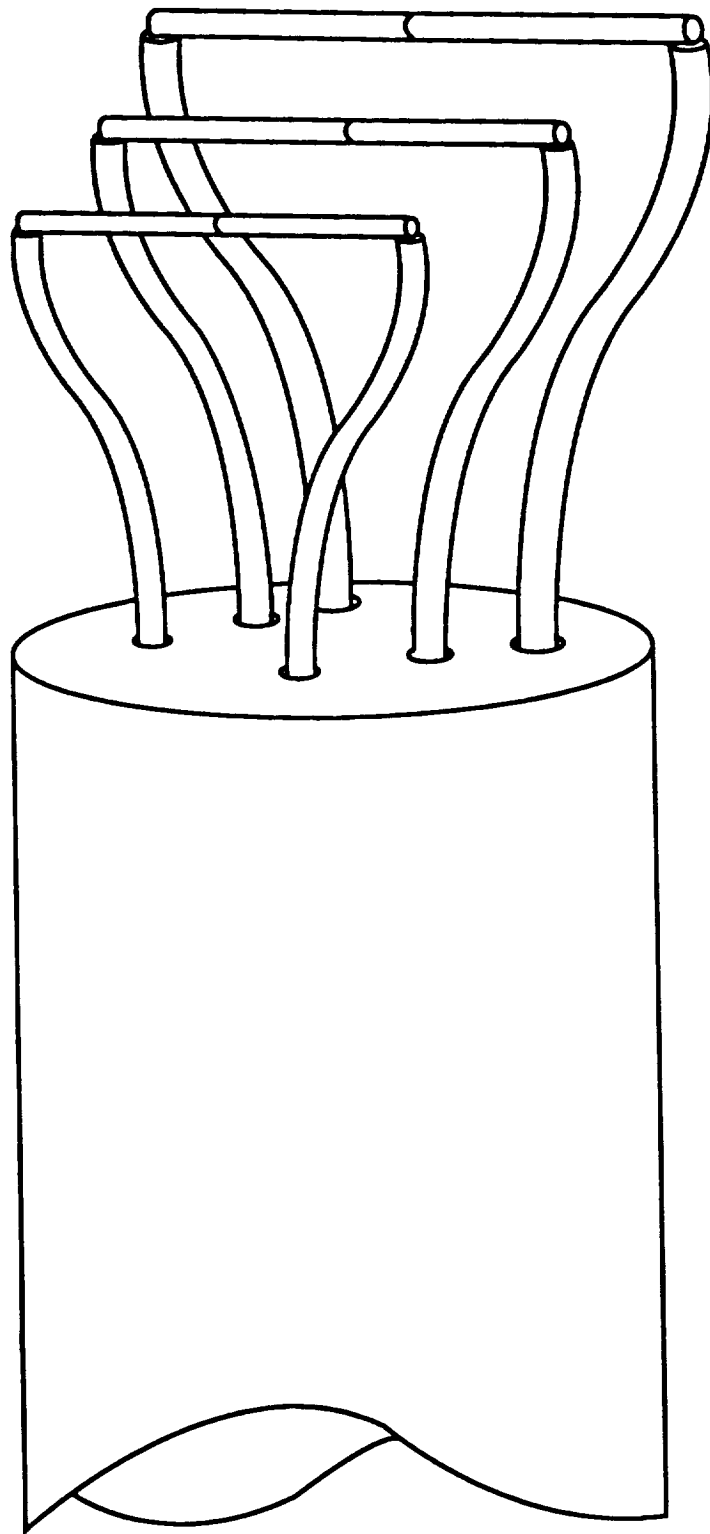
7. FIGURES

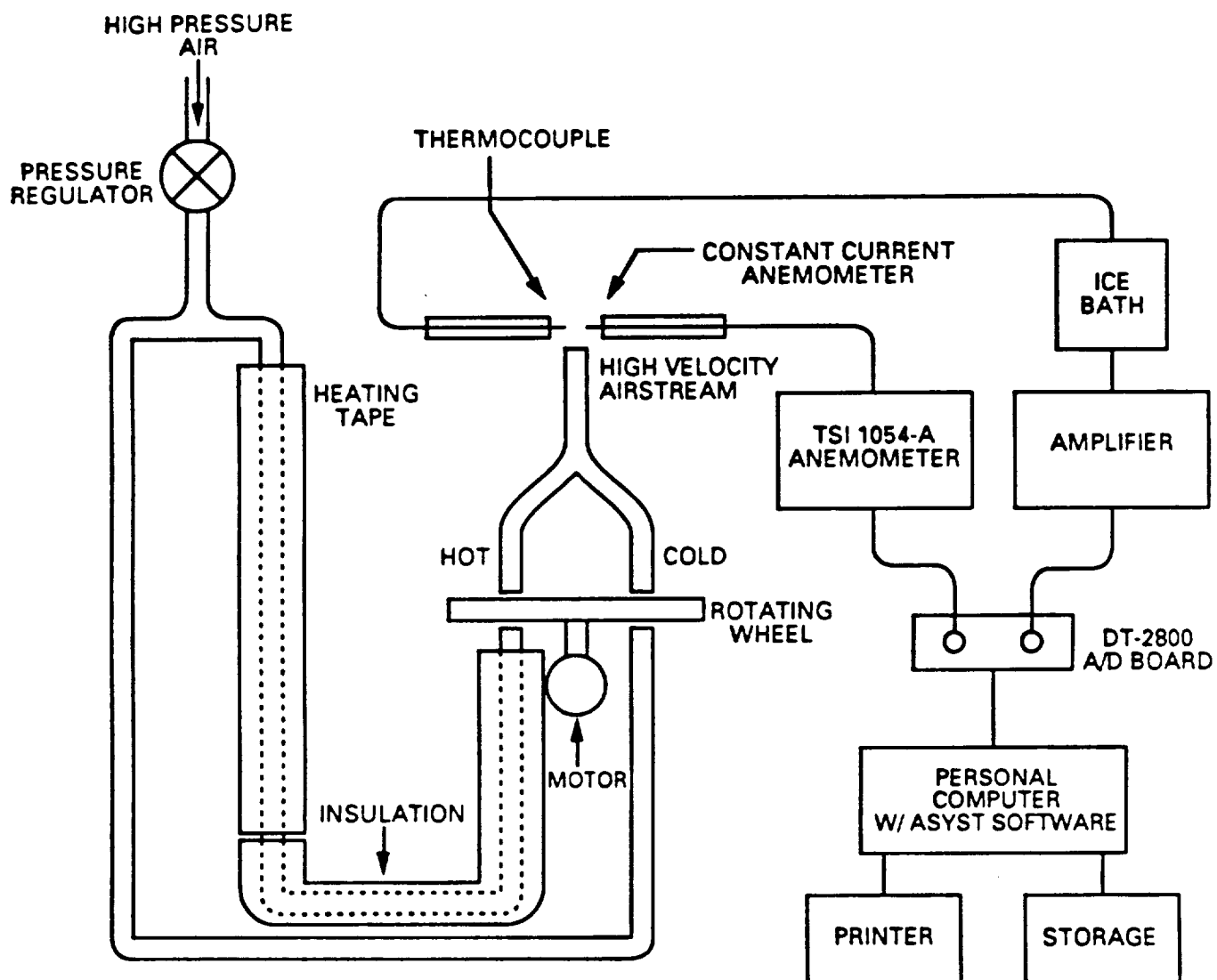Fig. 1 -   Schematic of three wire thermocouple.

Fig. 2 -   Schematic of rotating wheel apparatus.
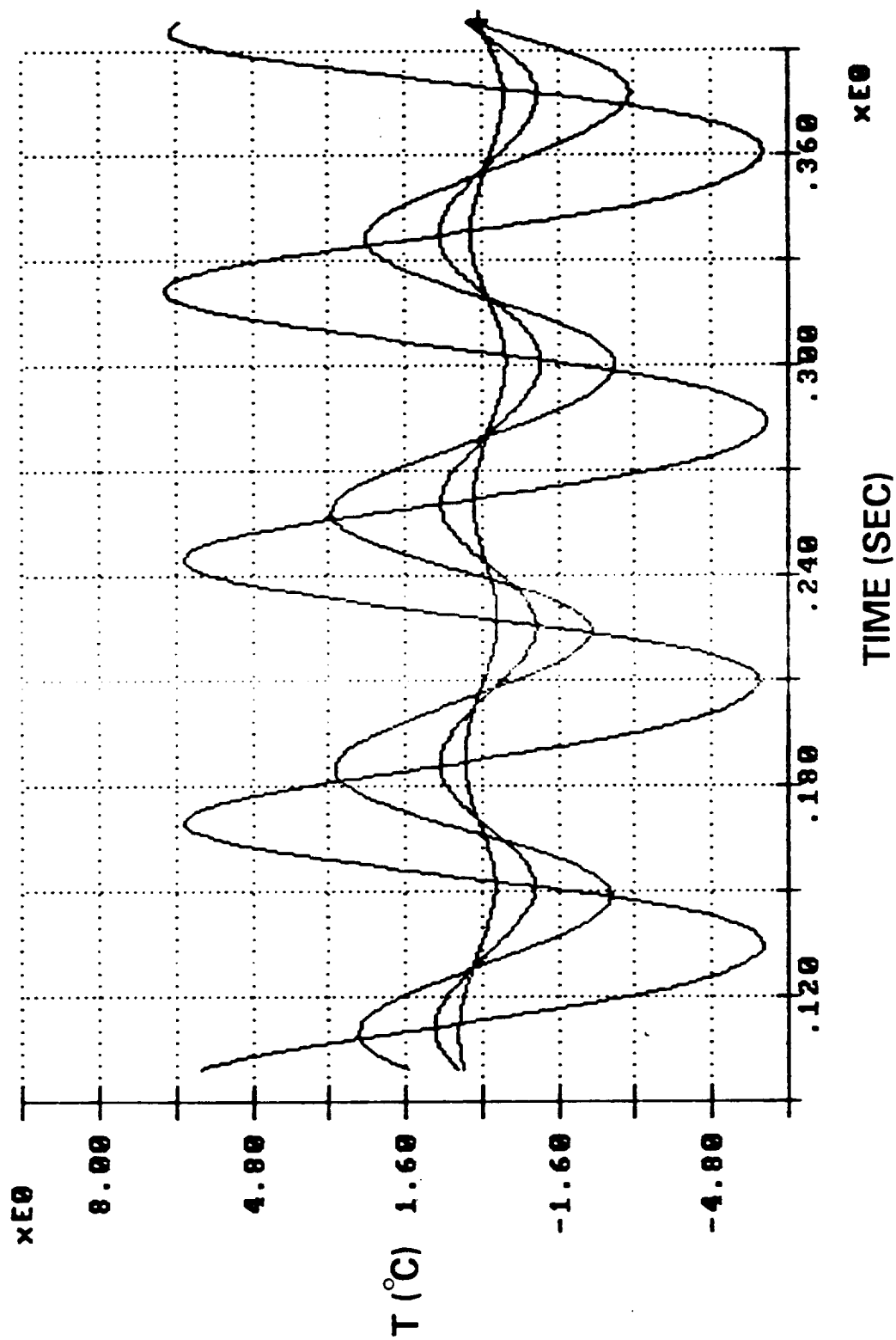
Fig. 3 -   Temperature profiles for the gas (large amplitude) and three type

K thermocouple wires.   Three wires: $D_1 = 50.8\mu m$, $\omega_1 = 42\ s^{-1}$, $D_2 =$

101.6μm, $D_3 = 203.2\mu m$.   Gas temperature frequency: $\omega = 86\ s^{-1}$.

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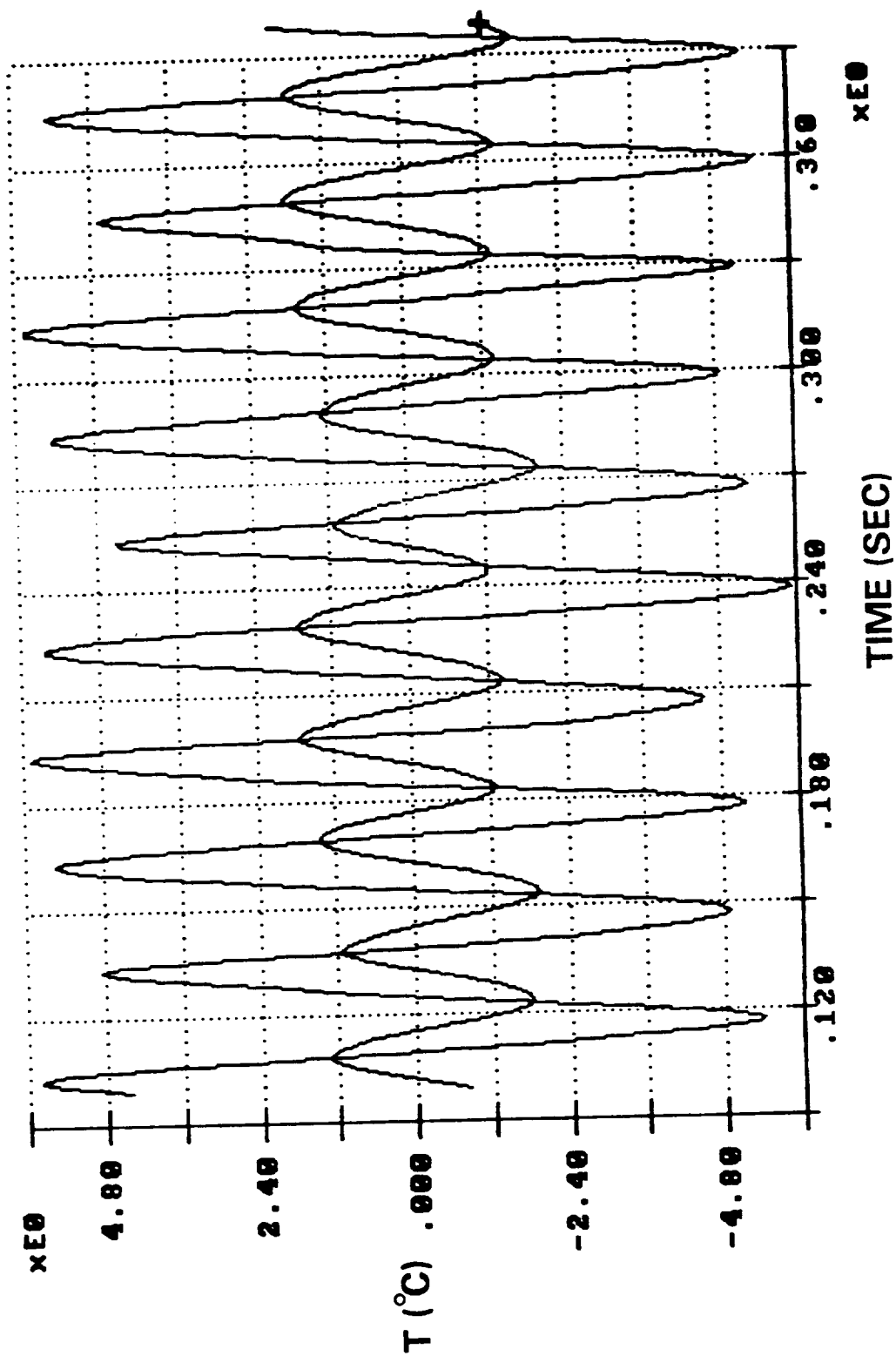
Fig. 4 - Gas temperature profile compared to reconstructed temperature derived from three wire thermocouple. $\omega_1 = 61$ s^{-1}, $\omega = 209$ s^{-1}, $\epsilon = -0.2$.

Fig. 5. - Same conditions as as fig. 4. $\in$ = 0.0.

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Fig. 6 -   Same conditoins as fig. 4. $\in$ = 0.2.

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Fig. 7 - Amplitude ratio of three wire thermocouple output-to-true gas temperature. Also shown is first order response of small wire.

III. MULTIWIRE THERMOCOUPLES IN REVERSING FLOW

1. INTRODUCTION

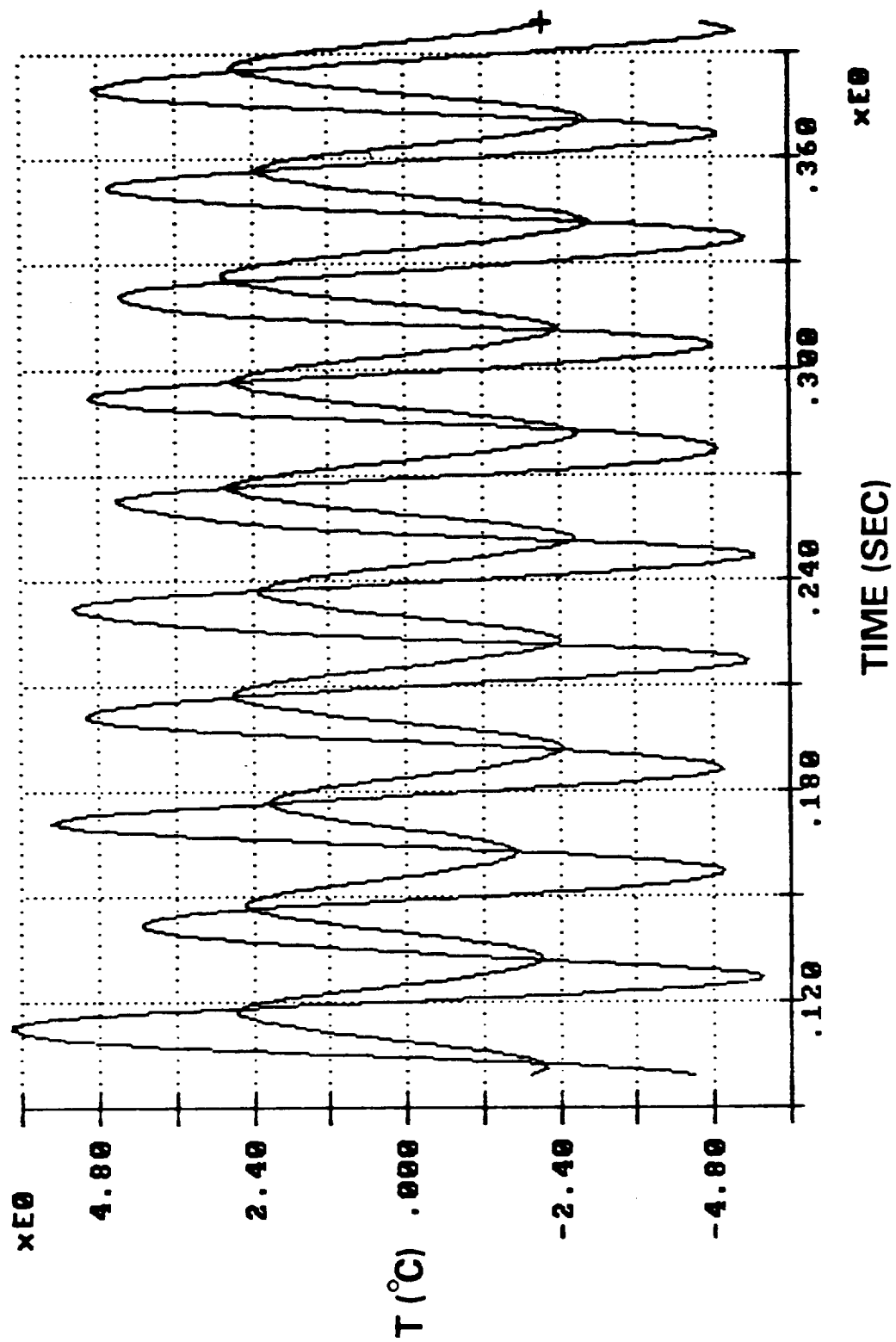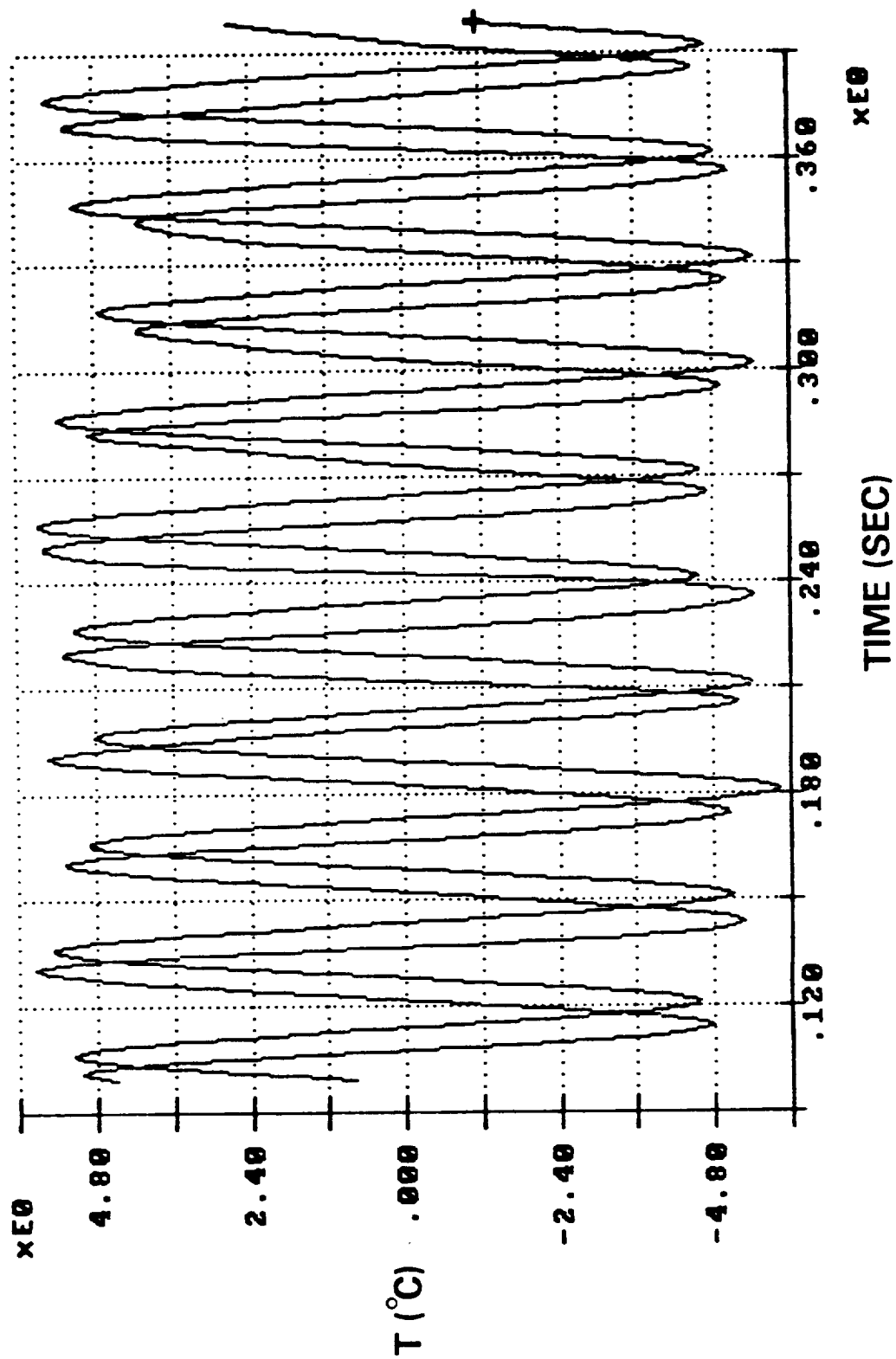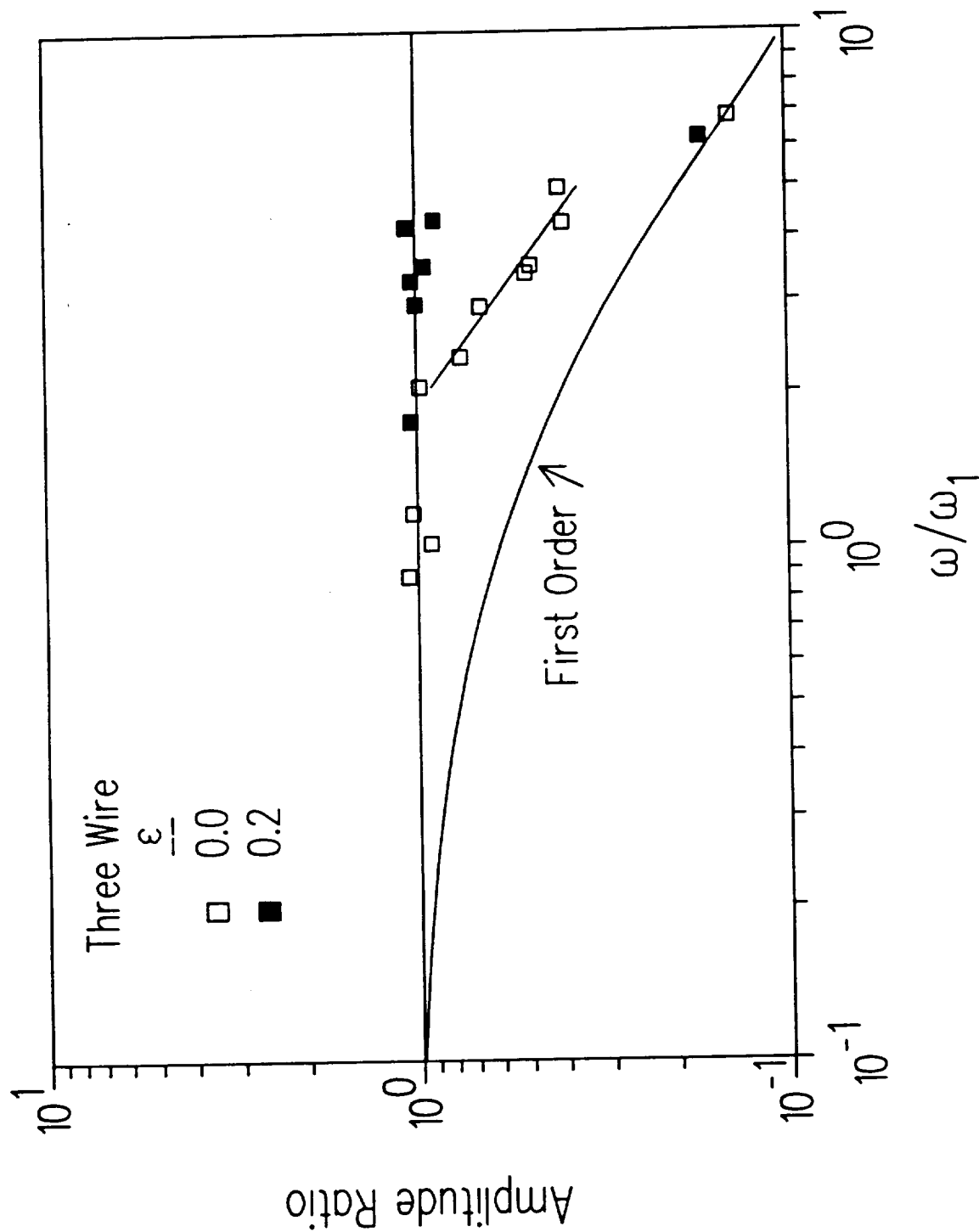The advantages of thermocouples are their low cost, reliability and simplicity since they do not require optical access or elaborate support electronics. The design of a thermocouple represents a compromise between accuracy, ruggedness and rapidity of response. Although thermocouples are suitable for the measurement of high frequency temperature fluctuations (< 1 KHz) in a flowing gas or liquid, the measured signal must be compensated since the frequency of the time-dependent fluid temperature can be much higher than the natural frequency of the thermocouple probe.[1] Moreover, use of a single wire thermocouple requires knowledge of the fluid velocity (normally assumed constant) and fluid properties (e.g., viscosity, density, etc.) to determine the convective heat transfer coefficient of the wire and its natural frequency.

The present paper describes the performance of novel two or three wire thermocouples with unequal wire diameters for use in unsteady or irregular flows of varying velocity. In this case the time constant or natural frequency of the thermocouple wire is time dependent and the normal procedure for compensation of high frequency signals is impractical. Previous work[3,4] indicated the usefulness of the concept in constant velocity flows where it was demonstrated that no compensation was required nor any knowledge of the fluid velocity or properties over a useful range of fluid temperature frequencies. In the present paper the multiwire thermocouple has been tested in a reversing flow field. The results of experimental measurements are presented along with the suggested procedure for the reduction of the data from the multiwire thermocouple as shown in fig. 1.

2. THEORY

Unsteady fluid motion develops in a variety of circumstances. For example, surfaces of discontinuity behind sharp edges such as airfoils and adverse pressure

gradients on blunt objects and in sharp corners lead to flow separation and the formation of recirculating eddies.[5] Other examples of irregular fluid motion are a.) cellular vorticies in stratified fluids or b.) oscillating flows in the exhaust from certain engines (e.g., stirling, etc.).

2.1 Natural Frequency of Single Wire

The conservation of energy for a single wire is :

$$\frac{dT}{dt} = \omega_n (T_g - T) \tag{1}$$

where the natural frequency

$$\omega_n = \frac{4h}{\rho c D}. \tag{2}$$

Here:

ω_n = natural frequency (sec^{-1})

T = temperature of the thermocouple ($^\circ$K)

T_g = temperature of the gas ($^\circ$K)

t = time (sec)

ρ = wire density (kg/m^3)

c = heat capacity of wire (J/kg - $^\circ$K)

h = heat transfer coefficient (W/m^2 - $^\circ$K)

D = wire diameter (m)

It is convenient to rewrite the heat transfer coefficient in terms of the Nusselt number

$$h = \frac{k_g}{D} Nu \tag{3}$$

where

k_g = thermal conductivity of the gas (W/m - $^\circ$K)

Nu = Nusselt number.

Thus, the natural frequency in Eq. (2) becomes

$$\omega_n = \frac{4 h_g \text{Nu}}{(\rho c) D^2},$$ (4)

where[6] $$\text{Nu} = C_o \text{Re}^m \text{Pr}^{1/3}.$$ (5)

and $$\text{Re} = \frac{|V(t)|D}{\upsilon_g}$$

$$\text{Pr} = \frac{\mu c_p}{k_g}$$

Here, Re and Pr are the Reynolds and Prandtl numbers, respectively and

μ = viscosity of gas (kg/m-sec)

u_g = Kinematic viscosity of gas (m^2 - sec^{-1}).

$V(t)$ = time dependent fluid velocity (m/s)

The natural frequency of a single wire from Eqs. (4) and (5) can now be written in the form,

$$\omega_n = \frac{k_g C_o 4 \text{Re}^m \text{Pr}^{1/3}}{(\rho c) D^2}$$ (6)

or separating the wire diameter and fluid velocity

$$\omega_n = C |V|^m D^{m-2}$$ (7)

where the coefficient C depends on both fluid and wire parameters or

$$C = \frac{4k_g C_o Pr^{1/3}}{(\rho c)\upsilon_g^m} \tag{8}$$

Substituting values corresponding to a type K thermocouple and a maximum gas velocity $V_{max} = 25$ m/sec^{-1} for air at standard conditions into Eqs. (7) and (8), one obtains values of the maximum natural frequency ω_{max}:

ω_{max}(sec^{-1})	D (m)	D(mil)
52.6	50.8x10-6	2
18.3	101.6x10-6	4
6.5	203.2x10-6	8

2.2. Two Wires

The conservation of energy given by Eq. (1) for a single wire contains two unknowns: ω_n the natural frequency and the desired gas temperature T_g. Each additional wire of a different diameter adds an unknown natural frequency ω_n. We also assume for a multiwire thermocouple that additional information is available concerning the ratios of natural frequencies. The latter provides a sufficient number of independent equations to determine the time-dependent gas temperature T_g.

Consider two thermocouple wires of unequal diameter $D_1 < D_2$. The conservation of energy for each wire becomes

$$\frac{dT_1}{dt} = C |V|^m D_1^{m-2}(T_g - T_1) \tag{9}$$

$$\frac{dT_2}{dt} = C |V|^m D_2^{m-2}(T_g - T_2). \tag{10}$$

In principle, knowledge of the wire diameters provides an additional equation

$$\frac{\omega_1}{\omega_2} = \left(\frac{D_1}{D_2}\right)^{m-2} \tag{11}$$

representing the ratio of natural frequencies as defined by Eq. (7). Dividing Eq. (9) by Eq. (11) and solving for the gas temperature one obtains

$$T_g = \frac{T_2\left(\dfrac{\dot{T}_1}{\dot{T}_2}\right) - T_1\left(\dfrac{D_1}{D_2}\right)^{m-2}}{\left(\dfrac{\dot{T}_1}{\dot{T}_2}\right) - \left(\dfrac{D_1}{D_2}\right)^{m-2}} \tag{12}$$

where $\dot{T}_1 = dT_1/dt$.

Our experience with multiwire thermocouples in constant flow[3,4] and the present study indicate that construction of the gas temperature from Eq. (12), which requires the measured wire signals T_1, T_2 and their derivatives, is unstable and unreliable. We have found in all cases, however, that satisfactory results are obtained by taking the Fourier transform of the conservation equations and deriving the gas temperature in terms of transformed variables. For the case of unsteady flow, independent knowledge of the gas velocity is therefore required.

Thus, taking the Fast Fourier Transform (FFT) of Eqs. (9) and (10) and dividing the expressions, the transform of the product of gas temperature and fluid velocity is obtained in the form

$$\overline{T}_{gv} = \frac{\overline{T}_{2v}\left(\dfrac{\overline{\dot{T}}_1}{\overline{\dot{T}}_2}\right) - \overline{T}_{1v}\left(\dfrac{D_1}{D_2}\right)^{m-2}}{\left(\dfrac{\overline{\dot{T}}_1}{\overline{\dot{T}}_2}\right) - \left(\dfrac{D_1}{D_2}\right)^{m-2}} \tag{13}$$

where $\overline{T}_{1v}$ = FFT($|V|^m T_1$) and $\overline{T}_{gv}$ = FFT ($|V|^m T_g$). For convenience we also define the parameter $\beta = \omega_1 / \omega_2$ or

$$\beta = \left(\frac{D_1}{D_2}\right)^{m-2}.$$ (14)

The inverse transform of $\overline{T}_{gv}$ provides the reconstructed gas temperature or

$$T_g = \frac{1}{|V|^m} FFT^{-1}\left[\overline{T}_{gv}\right]$$

2.3. Three Wires

Consider three thermocouple wires of unequal diameters $D_1 < D_2 < D_3$ as shown in fig. 1. The conservation expression for each wire becomes

$$\frac{dT_1}{dt} = \omega_1\left(T_g - T_1\right)$$ (15)

$$\frac{dT_2}{dt} = \omega_2\left(T_g - T_2\right)$$ (16)

$$\frac{dT_3}{dt} = \omega_3\left(T_g - T_3\right)$$ (17)

with the additional constraint

$$\alpha = \frac{\omega_2^2}{\omega_1 \omega_3} = \left(\frac{D_2^2}{D_1 D_3}\right)^{m-2}.$$ (18)

Taking the FFT of Eqs. (15)-(17), squaring Eq. (16) and dividing by the product of Eqs. (15) and (17), one obtains

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$$\left(\frac{\overline{T}_2^2}{\overline{T}_1 \overline{T}_3} \right) = \frac{\alpha \left(\overline{T}_{gv} - \overline{T}_{2v} \right)^2}{\left(\overline{T}_{gv} - \overline{T}_{1v} \right) \left(\overline{T}_{gv} - \overline{T}_{3v} \right)}. \tag{19}$$

Substituting $r = \overline{T}_2^2 / \overline{T}_3 \overline{T}_1$ and solving for $\overline{T}_{gv}$, one obtains

$$\overline{T}_{gv} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{20}$$

where
$$a = r - \alpha$$
$$b = 2\alpha \overline{T}_{2v} - r \left(\overline{T}_{1v} + \overline{T}_{3v} \right)$$
$$c = r \overline{T}_{1v} \overline{T}_{3v} - \alpha \overline{T}_{2v}^2$$

and $\overline{T}_{gv} = FFT \left(|V|^m T_g \right)$, $\overline{T}_{1v} = FFT \left(|V|^m T_1 \right)$. The inverse transform of $\overline{T}_{gv}$ provides the reconstructed gas temperature or

$$T_g = \frac{1}{|V|^m} FFT^{-1} \left[\overline{T}_{gv} \right] \tag{21}$$

3. EXPERIMENT

3.1 Apparatus

In the present experiment, thermocouple sensors are exposed to a reversing velocity air stream (<25 m/s) of varying temperature. In particular, the dynamic response of the thermocouple is measured for a periodic temperature profile of varying frequency. A rotating wheel configuration is used to deliver the test air stream to the proposed sensors. A similar experimental apparatus was described in detail by Elmore et al[7] and Forney et al[8].

A schematic of the rotating wheel apparatus used in the present experiment is shown in Fig. 2. As the wheel rotates, holes pass the two air supply tubes (3/4-in. i.d. copper) that allow slugs of hot (~55°C) and cold (~30°C) air to alternately enter the tube assembly mounted directly above the rotating wheel. The slugs of hot and cold air form a reversing air stream providing a periodic temperature profile covering a range of gas temperature frequencies from roughly 1 to 60 Hz.

The analog temperature signal is digitized with a Data Translation DT - 2801 A/D board mounted in an expansion slot of an IBM AT compatible computer as shown in Fig. 2. ASYST software was loaded onto the hard disk of the personal computer and this provided a flexible system for data storage, manipulation, and display. The true temperature profile of the airstream is measured with a constant current anemometer (TSI 1054-A) and sensor (1210-T1.5).

The three type K thermocouple wires were of diameter D_1 = 50.8μm (2mil), D_2 = 101.6μm and D_3 = 203.2μm as shown in fig. 1. The thermocouple junctions were fabricated by the Research Instrumentation Branch at the NASA Lewis Center. The wires were cut with a razor blade to produce a flat edge perpendicular to the wire axis. The wire segments were then mounted in a fixture such that the two faces of the junction were held together by springs. The junction was then formed by laser heating. The laser used was a KORAD model KWD Nd: YAG laser, operating at a wave length of 1060 nm. The power settings depend on the wire size and material, but for the thermocouple described in this report, a pulse duration of 4 mn was used delivering a total energy of approximately 2J.

3.2 Procedure

ASYST software was developed to acquire temperature data sequentially from the thermocouple and constant current anemometer. The data are digitized for four channels at a sampling rate of 512 Hz per channel for a total sample time of 0.5 s.

After data acquisition, the Fast Fourier Transform (FFT) is taken for each channel. The inverse transform of $\overline{T}_{gv}$ by the ASYST code provided a reconstruction of the true gas temperature.

4. RESULTS AND DISCUSSION

The velocity profile of the airstream was measured with the constant current anemometer. Figure 3 illustrates the velocity profile for a wheel frequency of 10 Hz. For the positive velocities in fig. 3 the gas reached a temperature of roughly 25 to 30°C corresponding to the cold slugs while the negative velocity components were hot air slugs of approximately 50°C.

In the present study the gas velocity and temperature were not measured simultaneously. Rather, it was assumed that the temperature measured with the anemometer was proportional to the gas velocity in the reversing flow field or V(t) α $T_g(t)$. A typical data profile is illustrated in fig. 4. The uniform profile on the right is the temperature profile measured with the anemometer. The temperature profile on the left was reconstructed with Eg. (20) assuming that the exponent m = 0.5 in Eqs. (7) and (21) and a value of the diameter ratio α = 1.0 for the three wire thermocouple.

The value of the exponent m = 0.5 was chosen for the Reynolds number that appears in the Nusselt number correlation Eq. (5) and the wire natural frequency wn given by Eq. (7). At the maximum gas velocity of 25 m/s the maximum Reynolds number for the three wires in the thermocouple assembly are 80, 160, 320 corresponding to the smallest to largest diameter wire, respectively. Since the recommended values of the exponent m are 0.5 for Re > 40 and 0.4 for $1 \le Re \le 40$,[6] we chose a constant value m = 0.5 in the present study.

4.1 Two Wires

The amplitude ratio of thermocouple-to-gas temperature was determined using a two-wire combination with $D_1 = 50.8$ μm and $D_2 = 101.6$ μm from the schematic in fig. 1. The gas temperature was reconstructed with Eq. (13) assuming m = 0.5. The amplitude ratio was determined by averaging peak temperatures over six or eight cycles. The results are indicated in fig. 5 as a function of the parameter $\beta = \omega_1/\omega_2 = (D_1/D_2)^{-3/2}$. In fig. (5) the natural frequency $\omega_1 = 59$ s^{-1} is defined as the frequency at the maximum gas velocity for the small wire $D_1 = 50.8$ μm. It is clear that by amplitude ratio is strongly affected by β and that the optimum value is $\beta = 2.55$.

Figure 6 illustrates amplitude attenuation as a function of gas temperature frequency for fixed $\beta = 2.55$. The two wire combination requires no compensation for frequencies $\omega/\omega_1 < 2.3$ corresponding to frequencies < 22 Hz in the reversing flow field.

4.2 Three Wires

The amplitude ratio of thermocouple-to-gas temperature was determined for the three wire combination as shown in the schematic of fig. 1. The gas temperature was reconstructed with Eqs. (20) and (21) with the exponent m = 0.5. The results are indicated in fig. 7 as a function of the parameter $\alpha=(\omega_2^2/\omega_1\omega_3)=(D_2^2/D_1D_3)^{-3/2}$. It is clear that the amplitude is strongly affected by a and that the optimum value is $\alpha = 1.0$.

Figure 8 illustrates amplitude attenuation as a function of gas temperature frequency for fixed a = 1.0. The three wire combination requires no compensation for frequencies $\omega/\omega_1 < 3.5$ corresponding to frequencies < 34 Hz in the reversing flow field.

5. CONCLUSIONS

Measurements are recorded for multiwire thermocouples consisting of either two or three wires of unequal diameters. Signals from the multiwire probe are recorded for a reversing gas flow with a periodic temperature fluctuation. It is demonstrated that the reconstructed signal from the multiwire thermocouple requires no compensation provided $\omega/\omega_1 < 2.3$ for two wires or $\omega/\omega_1 < 3.6$ for three wires where ω_1 is the natural frequency of the smaller wire based on the maximum gas velocity. The latter results were possible provided Fourier transformed data were used and knowledge of the gas velocity is available.

6. REFERENCES

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8. L. J. Forney, E. L. Meeks, J.Ma and G. C. Fralick, Measurement of Frequency Response in Short Thermocouple Wires, Rev. Sci. Instrum 64 3236 (1993).
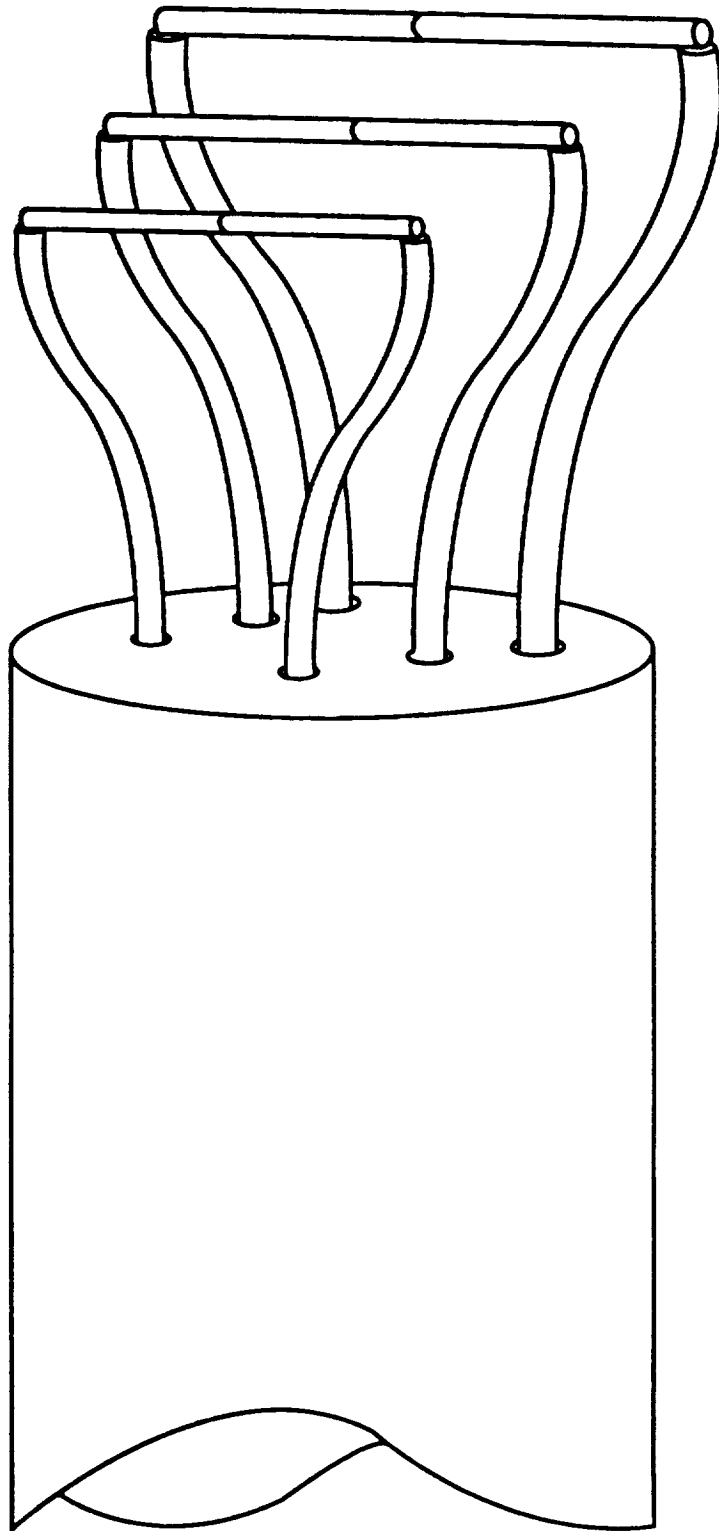
7. FIGURES

Figure 1   -   Schematic of three wire thermocouple
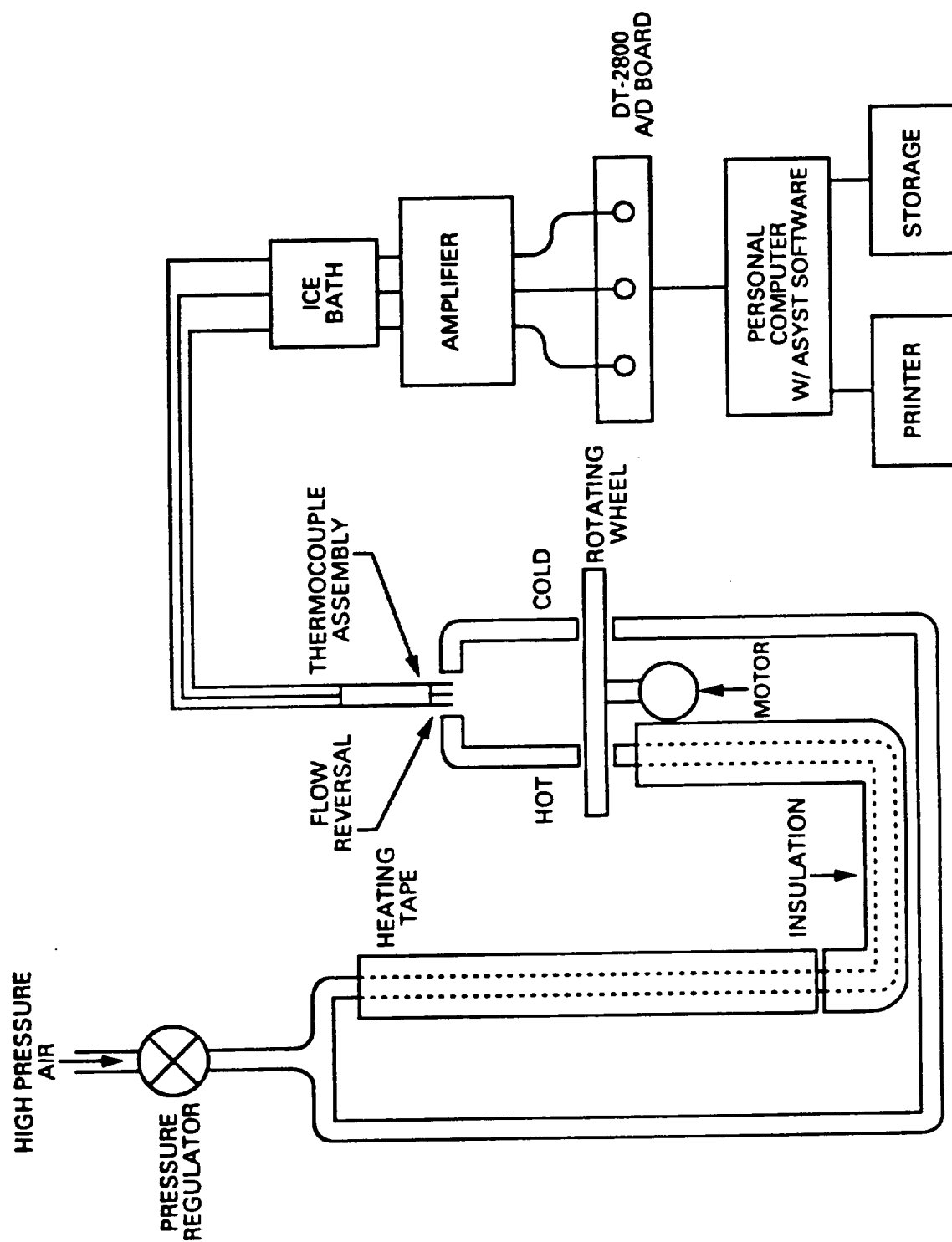
Figure 2   -   Schematic of rotating wheel apparatus.
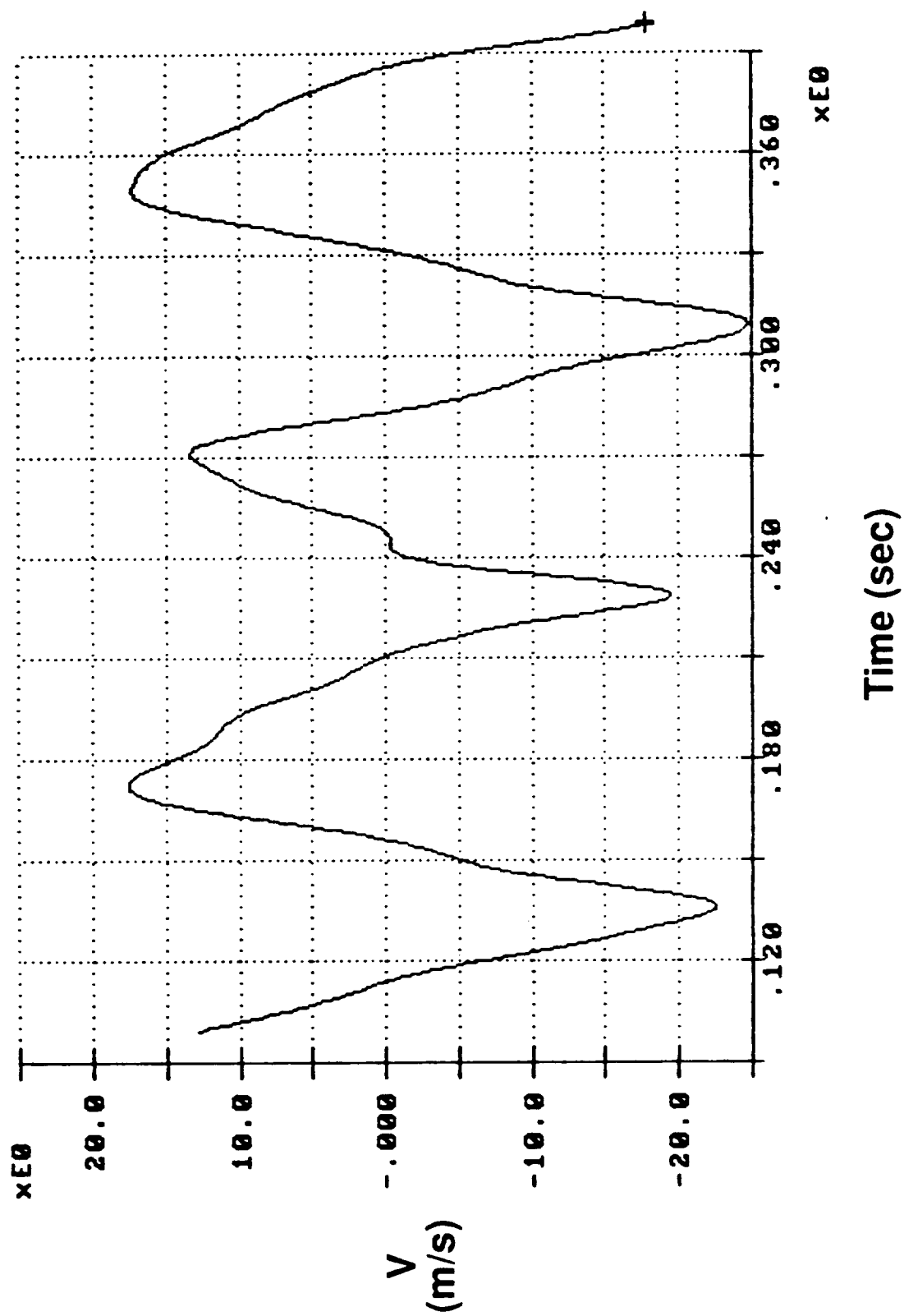
Figure 3   -   Velocity profile for reversing flow. Frequency of reversing flow f
=10 Hz.

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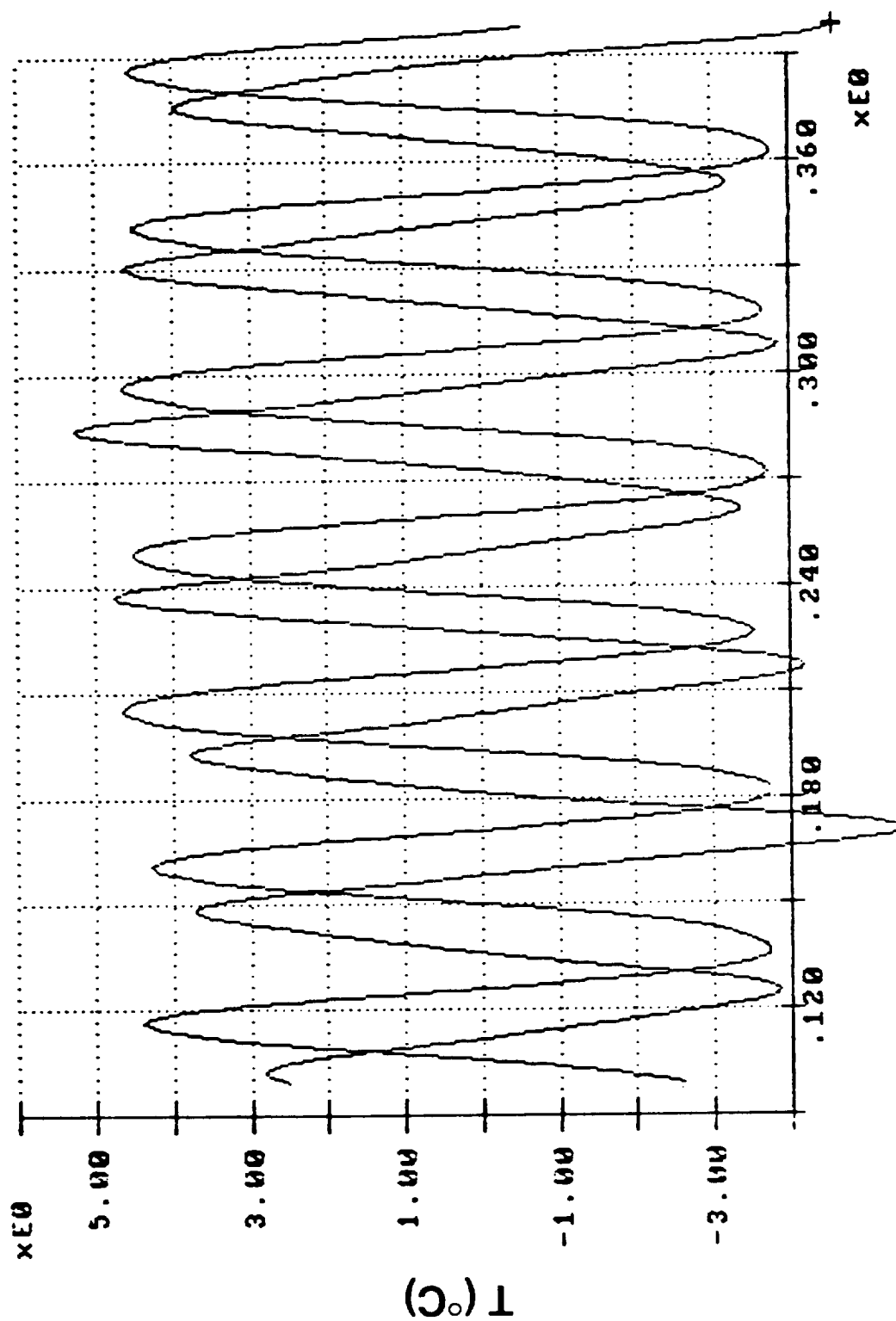
Figure 4 - Gas temperature profile compared to reconstructed temperature derived from three wire thermocouple. Frequency of temperature and flow reversal f = 21.5 Hz (ω = 135). Diameter ratio α = 1.0. The true gas temperature profile is shifted to the right.

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Figure 5   -   Amplitude ratio of two thermocouple output-to-true gas temperature. Nominal value for diameter ratio β_o = 2.83.
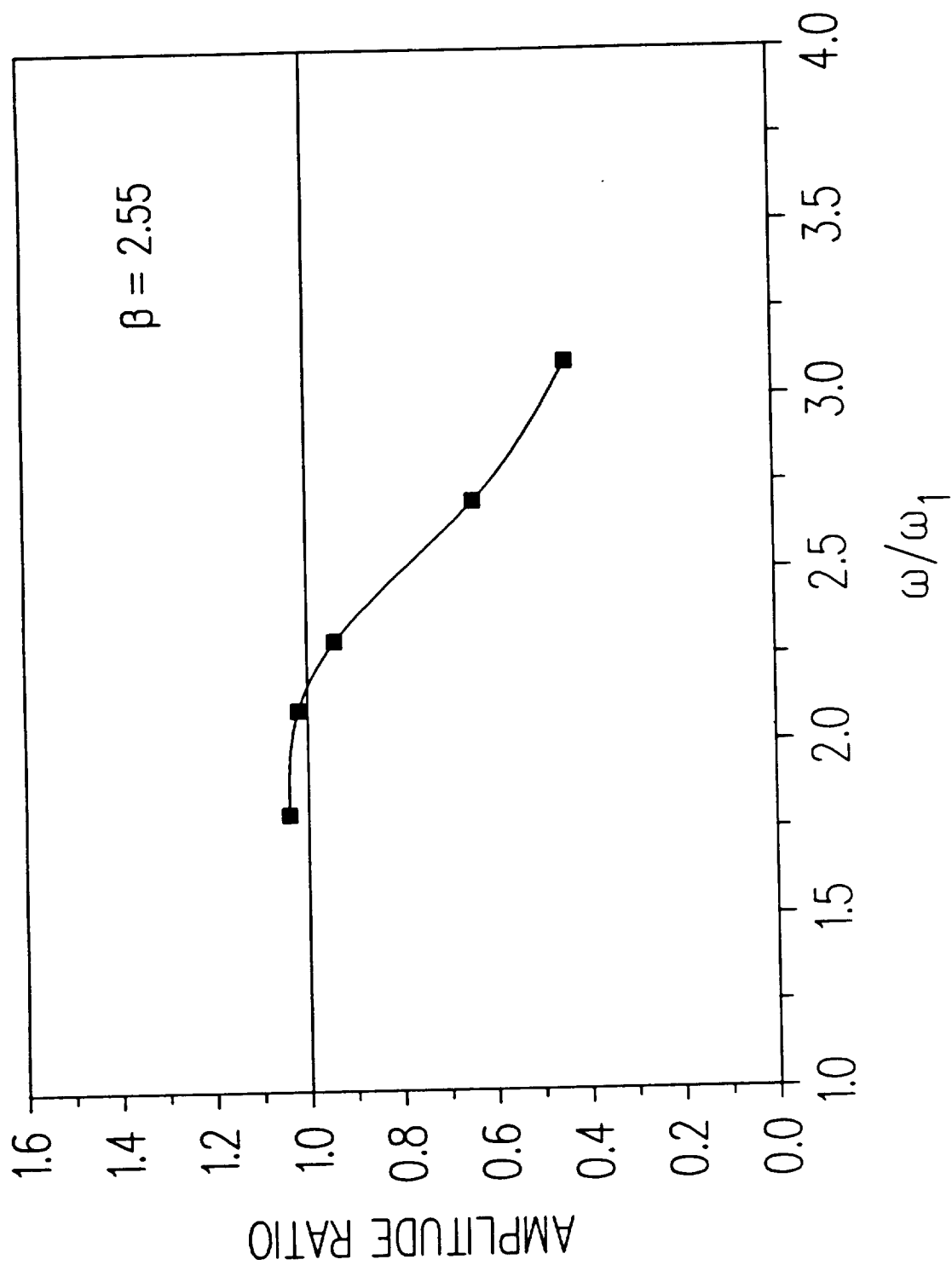
Figure 6 — Amplitude ratio of two thermocouple output versus gas temperature frequency of small wire based on maximum gas velocity. $\omega_1 = 59$ s^{-1}

Figure 7 - Amplitude ratio of three thermocouple output-to-true gas temperature. Nominal value for diameter ratio $\alpha_o = 1.0$.
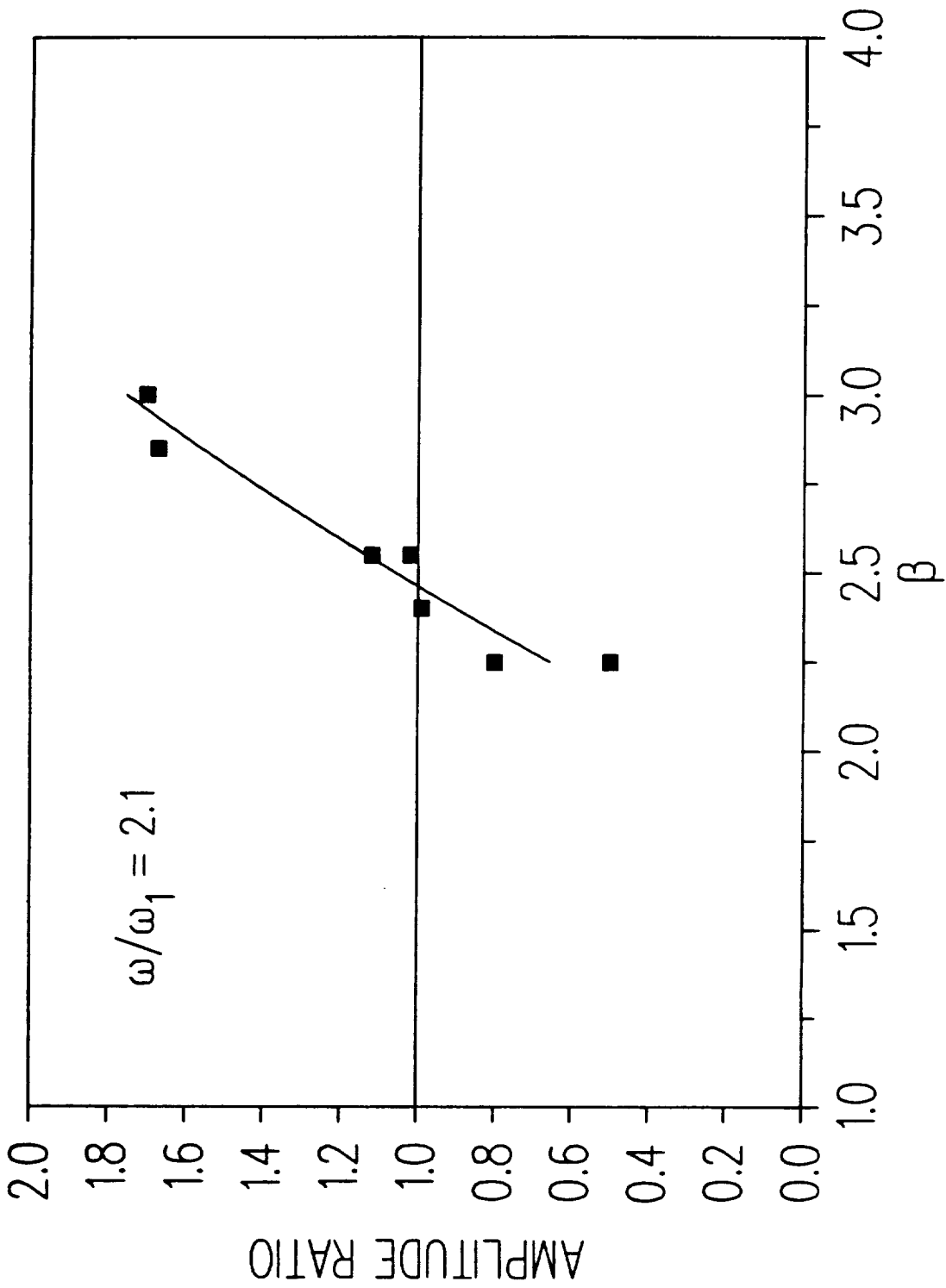
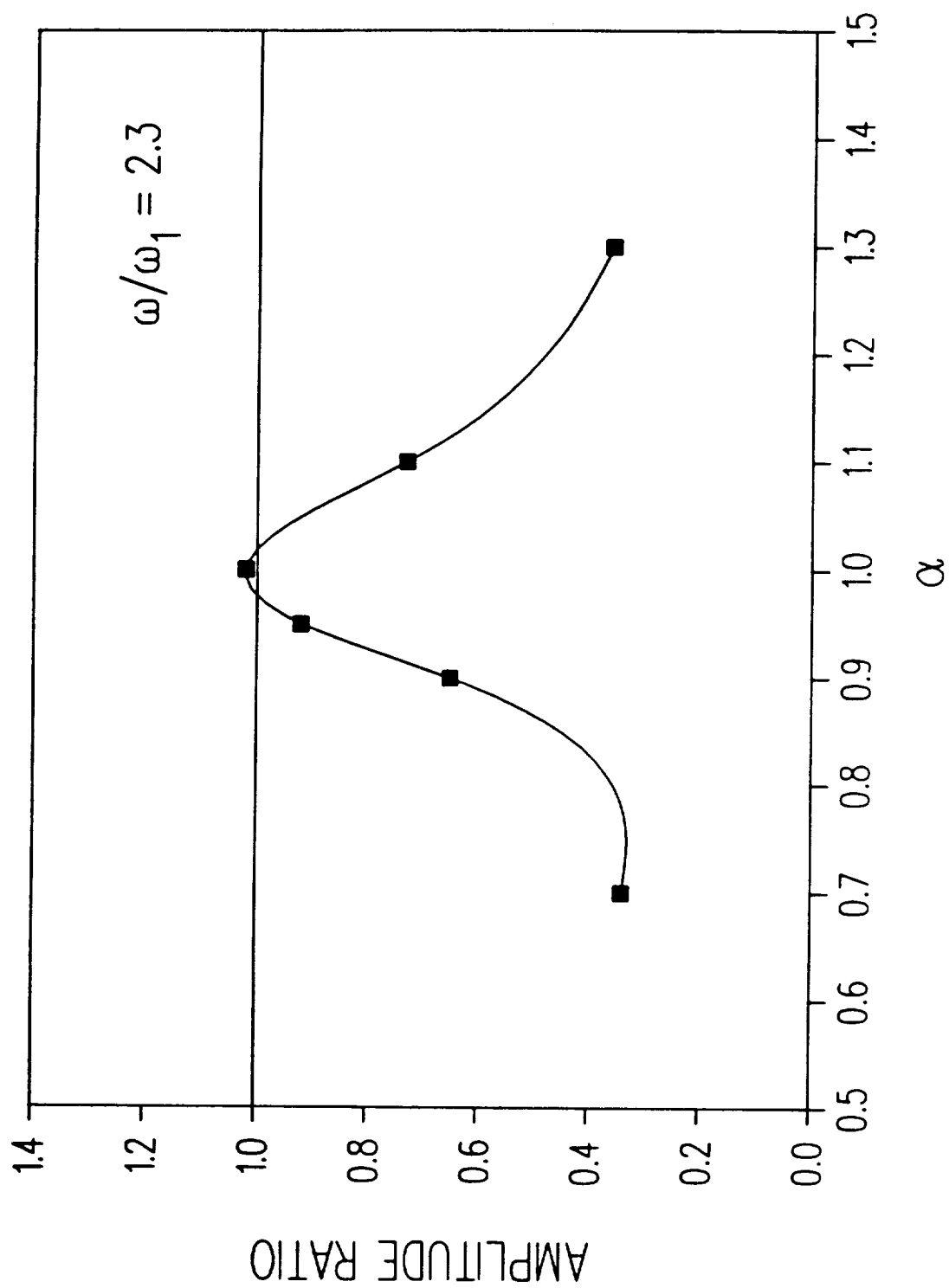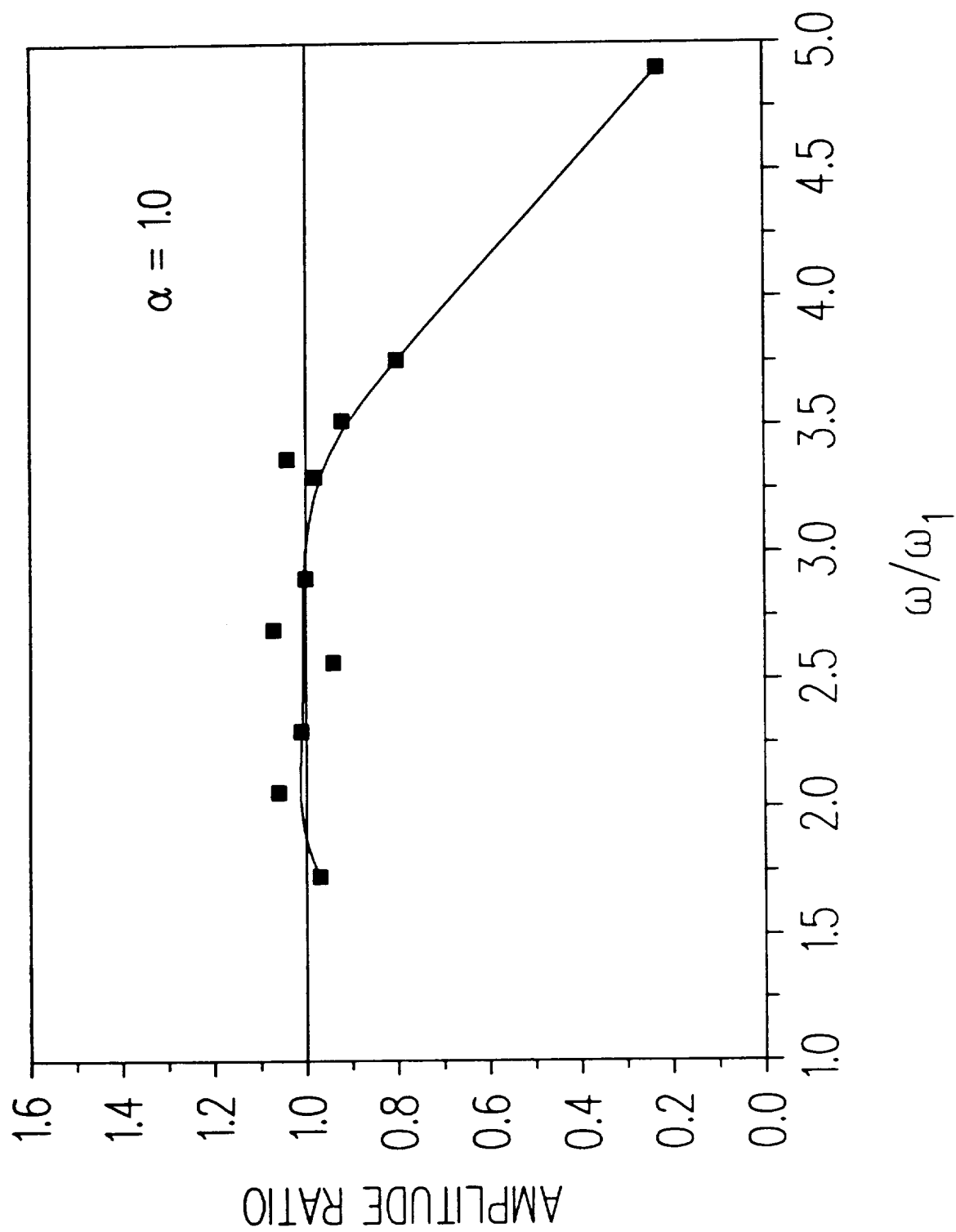Figure 8 - Amplitude ratio of three thermocouple output versus gas temperature frequency. $\omega_1 = 59\ s^{-1}$.

IV. APPENDIX - ASYST CODES

1. ASYST CODE - CONSTANT FLOW

```
\ FF3P.FOR
ECHO.OFF
FORGET.ALL
REAL DIM[ 512 ] ARRAY TIME
COMPLEX DIM[ 512 ] ARRAY TRANS1
COMPLEX DIM[ 512 ] ARRAY TRANS2
COMPLEX DIM[ 512 ] ARRAY TRANS3
REAL DIM[ 10 ] ARRAY ANS
REAL DIM[ 512 ] ARRAY ZMAGO
REAL DIM[ 512 ] ARRAY ZARGO
INTEGER SCALAR NUM
REAL SCALAR MEN
REAL SCALAR MEN1
REAL SCALAR MEN2
REAL SCALAR MEN3
REAL SCALAR MEN4
REAL SCALAR CD
REAL SCALAR TI
REAL DIM[ 512 ] ARRAY FREQS
REAL DIM[ 512 ] ARRAY SIGNAL
INTEGER DIM[ 512 , 4 ] ARRAY DATA.BUFFER



.25 CD :=
.0000  DATA.BUFFER :=
LOAD.OVERLAY ACQUIS.SOV
0  3  A/D.TEMPLATE DEMO.TEMPLATE     \ DEFINE AN A/D TEMPLATE
DATA.BUFFER TEMPLATE.BUFFER          \ DECLARE ARRAY AS A TEMPLATE BUFFER
CYCLIC                               \ SET TEMPLATE BUFFER TO CYCLIC MODE
512.   TEMPLATE.REPEAT               \ SET REPETITIONS FOR I/O INSTRUCTION
CD  CONVERSION.DELAY                 \ CONVERSION RATE (MSECS/SAMPLE)
DEMO.TEMPLATE A/D.INIT               \ INITIALIZE CURRENT A/D TEMPLATE
\  1. SYNC.PERIOD
\  SYNCHRONIZE
A/D.IN>ARRAY                         \ A/D INPUT TO TEMPLATES BUFFER
LOAD.OVERLAY WAVEOFS.SOV
512 REAL RAMP 1 - 4. * CD * 1000. / TIME :=        \ SET TIME AXIS


DATA.BUFFER  XSECT[ 1 ] 2048 -      \ CHANNEL 0 ON STACK (ANEMOMETER)
SIGNAL := SIGNAL
-.9336 ** 1324. *                    \ DEGREES CENTIGRADE
SIGNAL := SIGNAL
.05 SET.CUTOFF.FREQ                  \ SMOOTH DATA (CYCLES/POINT)
SMOOTH  SIGNAL := SIGNAL
MEAN MEN := SIGNAL
MEN - SIGNAL :=                      \ CENTER ON ORIGIN
TIME SUB[ 100 , 300 , 1 ]  SIGNAL SUB[ 100 , 300 , 1 ]
XY.AUTO.PLOT                         \ PLOT CHANNEL 0

SIGNAL
```

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```
FFT                                           \ TAKE FFT
TRANS1 := TRANS1 ZMAG ZMAGO :=                \ MAGNITUDE OF FFT
5 SET.#.POINTS
1 SET.#.OPTIMA
ZMAGO SUB[ 1 , 120 , 1 ] LOCAL.MAXIMA              \ FIND INDEX AND MAX OF MAGNITUDE
SWAP NUM := NUM                                    \ INDEX OF MAX MAGNITUDE
1 - CD / 4. / 512. / 1000. *  PI * 2. *            \ FREQUENCY AT MAX MAGNITUDE
ANS [ 1 ] := ANS [ 1 ] DROP ANS [ 2 ] :=
DEG TRANS1 ZARG ZARGO := ZARGO [ NUM ]             \ FIND ARGUMENT OF FFT
ANS [ 3 ] :=                                       \ ARGUMENT AT MAX MAGNITUDE
TRANS1 [ NUM ] DEG ZARG   ANS [ 3 ] :=




DATA.BUFFER XSECT[ 2 ] 2048 -                 \ CHANNEL 1 ON STACK (THERMOCOUPLE)
SIGNAL := SIGNAL
.04725 *
SIGNAL := SIGNAL

\ SMOOTH  SIGNAL := SIGNAL                    \ SMOOTH DATA (CYCLES/POINT)
MEAN MEN1 :=     SIGNAL MEN1 - SIGNAL :=
TIME SUB[ 100 , 300 , 1 ]  SIGNAL SUB[ 100 , 300 , 1 ]
XY.DATA.PLOT                                  \ PLOT CHANNEL 1

SIGNAL
FFT                                           \ TAKE FFT
TRANS1 := TRANS1 ZMAG ZMAGO :=                \ MAGNITUDE OF FFT
ZMAGO SUB[ 1 , 120 , 1 ] LOCAL.MAXIMA              \ FIND INDEX AND MAX OF MAGNITUDE
SWAP NUM := NUM                                    \ INDEX OF MAX MAGNITUDE
1 - CD / 4. / 512. / 1000. * PI * 2. *             \ FREQUENCY AT MAX MAGNITUDE
ANS [ 4 ] := ANS [ 4 ] DROP ANS [ 5 ] :=
DEG TRANS1 ZARG ZARGO := ZARGO [ NUM ]             \ FIND ARGUMENT OF FFT
ANS [ 6 ] :=                                       \ ARGUMENT AT MAX MAGNITUDE
TRANS1 [ NUM ] DEG ZARG   ANS [ 6 ] :=




DATA.BUFFER XSECT[ 3 ] 2048 -                 \ CHANNEL 2 ON STACK (THERMOCOUPLE)
SIGNAL := SIGNAL
.05457 *
SIGNAL := SIGNAL

\ SMOOTH  SIGNAL := SIGNAL                    \ SMOOTH DATA (CYCLES/POINT)
MEAN MEN2 :=     SIGNAL MEN2 - SIGNAL :=      \ CENTER ON ORIGIN
TIME SUB[ 100 , 300 , 1 ]  SIGNAL SUB[ 100 , 300 , 1 ]
XY.DATA.PLOT                                  \ PLOT CHANNEL 2

SIGNAL
FFT
TRANS2 :=                                     \ TRANSFORM OF CHANNEL 2
```

```
DATA.BUFFER XSECT[ 4 ] 2048 -              \ CHANNEL 3 ON STACK (THERMOCOUPLE)
SIGNAL := SIGNAL
.04725 *
SIGNAL := SIGNAL

\  SMOOTH  SIGNAL := SIGNAL               \ SMOOTH DATA (CYCLES/POINT)
MEAN MEN3 :=    SIGNAL MEN3 - SIGNAL :=   \ CENTER ON ORIGIN
TIME SUB[ 100 , 300 , 1 ]  SIGNAL SUB[ 100 , 300 , 1 ]
XY.DATA.PLOT                              \ PLOT CHANNEL 3


SIGNAL                                    \ TRANSFORM OF CHANNEL 3
FFT
TRANS3 :=


   TRANS2 TRANS1 * TRANS2 TRANS3 * +      \ CONSTRUCT SIGNAL
   2. TRANS1 * TRANS3 * - TRANS2 *
   TRANS2 TRANS2 * TRANS1 TRANS3 * - /
   TRANS1 :=

\   TRANS1 ZMAG ZMAG0 :=                  \ AMPLITUDE OF FIRST HARMONIC
\   ZMAG0 SUB[ 1 , 120 , 1 ] LOCAL.MAXIMA
\   ANS [ 7 ] :=

TRANS1                                    \ INVERSE TRANSFORM
IFFT
ZREAL SIGNAL :=  SIGNAL
   MEAN MEN4 := SIGNAL MEN4 - SIGNAL := SIGNAL
   SMOOTH SIGNAL :=
SIGNAL    SIGNAL :=
TIME SUB[ 100 , 300 , 1 ]
SIGNAL SUB[ 100 , 300 , 1 ]
XY.DATA.PLOT                              \ PLOT RECONSTRUCTED SIGNAL

CR
\   ."   FREQ      MAG     PHI  " CR
   ANS [ 1 ]   ? CR                       \ ANGULAR FREQ CHANNEL 0
   ANS [ 5 ]  ANS [ 2 ] /  ? CR           \ AMPLITUDE RATIO CHANNELS 1/0
\  ANS [ 7 ] ?  ANS [ 2 ] ?  CR
\  ANS [ 7 ] ANS [ 2 ] / ? CR
\  ANS [ 3 ] -1 * ANS [ 6 ] +  180 PI / CD * 1000 / ANS [ 1 ] * -  ? CR
\  ANS [ 3 ] -1 * ANS [ 6 ] + ? CR
   MEN ? MEN1 ? MEN2 ? MEN3 ? MEN4 ? CR
\  FORGET.ALL
```

2. ASYST CODE - REVERSING FLOW

```
\ REV.FOR
ECHO.OFF
FORGET.ALL
REAL DIM[ 512 ] ARRAY TIME
COMPLEX DIM[ 512 ] ARRAY TRANS1
REAL DIM[ 512 ] ARRAY SIG1
REAL DIM[ 512 ] ARRAY SIG2
REAL DIM[ 512 ] ARRAY SIG3
REAL DIM[ 512 ] ARRAY DIF1
REAL DIM[ 512 ] ARRAY DIF2
REAL DIM[ 512 ] ARRAY DIF3
REAL DIM[ 512 ] ARRAY R1
REAL DIM[ 10 ] ARRAY ANS
REAL DIM[ 512 ] ARRAY ZMAGO
REAL DIM[ 512 ] ARRAY ZARGO
INTEGER SCALAR NUM
REAL SCALAR MEN
REAL SCALAR MEN1
REAL SCALAR MEN2
REAL SCALAR MEN3
REAL SCALAR MEN4
REAL SCALAR CD
REAL SCALAR AL
REAL DIM[ 512 ] ARRAY SIGNAL
INTEGER DIM[ 512 , 4 ] ARRAY DATA.BUFFER


.25 CD :=
.0000  DATA.BUFFER :=
LOAD.OVERLAY ACQUIS.SOV
0  3  A/D.TEMPLATE DEMO.TEMPLATE    \ DEFINE AN A/D TEMPLATE
DATA.BUFFER TEMPLATE.BUFFER         \ DECLARE ARRAY AS A TEMPLATE BUFFER
CYCLIC                              \ SET TEMPLATE BUFFER TO CYCLIC MODE
512.   TEMPLATE.REPEAT              \ SET REPETITIONS FOR I/O INSTRUCTION
CD  CONVERSION.DELAY                \ CONVERSION RATE (MSECS/SAMPLE)
DEMO.TEMPLATE A/D.INIT              \ INITIALIZE CURRENT A/D TEMPLATE
\  1. SYNC.PERIOD
\  SYNCHRONIZE
A/D.IN>ARRAY                        \ A/D INPUT TO TEMPLATES BUFFER
LOAD.OVERLAY WAVEOPS.SOV
512 REAL RAMP 1 - 4. * CD * 1000. / TIME :=       \ SET TIME AXIS


DATA.BUFFER  XSECT[ 1 ] 2048 -     \ CHANNEL 0 ON STACK (ANEMOMETER)
SIGNAL := SIGNAL
-.9336 ** 1324. *                  \  DEGREES  CENTIGRADE
SIGNAL := SIGNAL
.1 SET.CUTOFF.FREQ                 \ SMOOTH DATA (CYCLES/POINT)
SMOOTH  SIGNAL := SIGNAL
MEAN MEN := SIGNAL
MEN - SIGNAL :=                    \ CENTER ON ORIGIN
```

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```
TIME SUB[ 100 , 300 , 1 ]  SIGNAL SUB[ 100 , 300 , 1 ]
XY.AUTO.PLOT                        \ PLOT CHANNEL 0

SIGNAL
FFT                                  \ TAKE FFT
TRANS1 := TRANS1 ZMAG ZMAG0 :=       \ MAGNITUDE OF FFT
5 SET.#.POINTS
1 SET.#.OPTIMA
ZMAG0 SUB[ 1 , 120 , 1 ] LOCAL.MAXIMA   \ FIND INDEX AND MAX OF MAGNITUDE
SWAP NUM := NUM                         \ INDEX OF MAX MAGNITUDE
1 - CD / 4. / 512. / 1000. * PI * 2. *  \ FREQUENCY AT MAX MAGNITUDE
ANS [ 1 ] := ANS [ 1 ] DROP ANS [ 2 ] :=
DEG TRANS1 ZARG ZARG0 := ZARG0 [ NUM ]  \ FIND ARGUMENT OF FFT
ANS [ 3 ] :=                            \ ARGUMENT AT MAX MAGNITUDE
TRANS1 [ NUM ] DEG ZARG   ANS [ 3 ] :=



DATA.BUFFER XSECT[ 2 ] 2048 -        \ CHANNEL 1 ON STACK (THERMOCOUPLE)
SIG1 := SIG1
.04725 *
SIG1 := SIG1

\  SMOOTH  SIG1 := SIG1             \ SMOOTH DATA (CYCLES/POINT)
MEAN MEN1 :=    SIG1 MEN1 - SIG1 :=
TIME SUB[ 100 , 300 , 1 ]  SIG1 SUB[ 100 , 300 , 1 ]
\   XY.DATA.PLOT                    \ PLOT CHANNEL 1

SIG1
FFT                                  \ TAKE FFT
TRANS1 := TRANS1 ZMAG ZMAG0 :=       \ MAGNITUDE OF FFT
ZMAG0 SUB[ 1 , 120 , 1 ] LOCAL.MAXIMA   \ FIND INDEX AND MAX OF MAGNITUDE
SWAP NUM := NUM                         \ INDEX OF MAX MAGNITUDE
1 - CD / 4. / 512. / 1000. * PI * 2. *  \ FREQUENCY AT MAX MAGNITUDE
ANS [ 4 ] := ANS [ 4 ] DROP ANS [ 5 ] :=
DEG TRANS1 ZARG ZARG0 := ZARG0 [ NUM ]  \ FIND ARGUMENT OF FFT
ANS [ 6 ] :=                            \ ARGUMENT AT MAX MAGNITUDE
TRANS1 [ NUM ] DEG ZARG   ANS [ 6 ] :=

SIG1
DIFFERENTIATE.DATA    .00001 +   DIF1 :=    \ DIFFERENTIATE CHANNEL 1



DATA.BUFFER XSECT[ 3 ] 2048 -        \ CHANNEL 2 ON STACK (THERMOCOUPLE)
SIG2 := SIG2
.05457 *
SIG2 := SIG2

SMOOTH  SIG2 := SIG2                 \ SMOOTH DATA (CYCLES/POINT)
MEAN MEN2 :=    SIG2 MEN2 - SIG2 :=  \ CENTER ON ORIGIN
TIME SUB[ 100 , 300 , 1 ]  SIG2 SUB[ 100 , 300 , 1 ]


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```

```
\   XY.DATA.PLOT                          \ PLOT CHANNEL 2

SIG2
DIFFERENTIATE.DATA      .00001 +    DIF2 :=    \ DIFFERENTIATE CHANNEL 2



DATA.BUFFER XSECT[ 4 ] 2048 -          \ CHANNEL 3 ON STACK (THERMOCOUPLE)
SIG3 := SIG3
.04725 *
SIG3 := SIG3

 SMOOTH  SIG3 := SIG3                        \ SMOOTH DATA (CYCLES/POINT)
MEAN MEN3 :=    SIG3 MEN3 - SIG3 :=          \ CENTER ON ORIGIN
TIME SUB[ 100 , 300 , 1 ] SIG3 SUB[ 100 , 300 , 1 ]
    \   XY.DATA.PLOT                         \ PLOT CHANNEL 3

SIG3
DIFFERENTIATE.DATA      .00001 +    DIF3 :=    \ DIFFERENTIATE CHANNEL 3


 1.02 AL :=
DIF2 DIF2 * DIF1 DIF3 * /    R1 :=           \ RECONSTRUCT SIGNAL
R1  AL -    DIF1 :=
2. SIG2 * AL * SIG1 SIG3 + R1 * -    DIF2 :=
SIG1 SIG3 * R1 * SIG2 SIG2 * AL * -    DIF3 :=
DIF2 DIF2 * 4. DIF1 * DIF3 * -   ABS   SQRT 1. *    DIF3 :=
DIF2 -1. *   DIF3 - 2. DIF1 * /   SIG1 :=


\    TRANS1 ZMAG ZMAG0 :=                    \ AMPLITUDE OF FIRST HARMONIC
\    ZMAG0 SUB[ 1 , 120 , 1 ] LOCAL.MAXIMA
\    ANS [ 7 ] :=

SIG1
MEAN MEN4 := SIG1 MEN4 - SIG1 := SIG1
   SMOOTH SIG1 :=
SIG1   SIG1 :=
TIME SUB[ 100 , 300 , 1 ]
SIG1 SUB[ 100 , 300 , 1 ]
   XY.DATA.PLOT                              \ PLOT RECONSTRUCTED SIGNAL

CR
\ ."    FREQ     MAG      PHI " CR
 ANS [ 1 ]   ? CR                            \ ANGULAR FREQ CHANNEL 0
 ANS [ 5 ] ANS [ 2 ] / ? CR                  \ AMPLITUDE RATIO CHANNELS 1/0
\ ANS [ 7 ] ? ANS [ 2 ] ? CR
\ ANS [ 7 ] ANS [ 2 ] / ? CR
\ ANS [ 3 ] -1 * ANS [ 6 ] + 180 PI / CD * 1000 / ANS [ 1 ] * - ? CR
\ ANS [ 3 ] -1 * ANS [ 6 ] + ? CR
MEN ? MEN1 ? MEN2 ? MEN3 ? MEN4 ? CR
\ FORGET.ALL
```

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